

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

1. (10 pts) For each matrix below, determine if it is invertible. If so, compute its inverse.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}.$$

Solution. A is not invertible since it is not square.

We find the reduced row echelon form of $[B \mid I_{2 \times 2}]$:

$$\begin{aligned} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{array} \right] &\rightarrow R_2 + R_1 \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array} \right] \rightarrow \frac{1}{2}R_2 \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array} \right] \\ &\rightarrow R_1 - 3R_2 \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array} \right]. \end{aligned}$$

Since the reduced row echelon form of B is I , B is invertible and $B^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$.

We find the reduced row echelon form of C :

$$\begin{aligned} \left[\begin{array}{ccc} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{array} \right] &\rightarrow \text{swap } R_1, R_3 \left[\begin{array}{ccc} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{array} \right] \rightarrow \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc} 1 & 2 & 0 \\ 0 & -3 & 3 \\ 0 & -1 & 1 \end{array} \right] \\ &\rightarrow \text{swap } R_2, R_3 \left[\begin{array}{ccc} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & -3 & 3 \end{array} \right] \rightarrow -R_2 \left[\begin{array}{ccc} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -3 & 3 \end{array} \right] \\ &\rightarrow \begin{array}{l} R_1 - 2R_2 \\ R_3 + 3R_2 \end{array} \left[\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array} \right]. \end{aligned}$$

Since the reduced row echelon form of C is not the identity $I_{3 \times 3}$, C is not invertible. □