Math 314 Fall 2007 **Quiz #2** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely. Your answer to each question below should be of the form "Yes, because..." or "No, because...."

Let $S = \left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \right\}$.

1. (5 pts) Is $\bar{v} = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}$ in span(S)?

   Solution. The vector $\bar{v}$ is in span(S) if there exist scalars c_1, c_2, c_3 such that

   $$c_1 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}.$$

   We solve by finding the reduced row echelon form of the corresponding augmented matrix:

   $$\begin{bmatrix} 2 & 3 & 1 & 3 \\ 2 & 1 & 3 & 7 \\ 1 & 2 & 0 & 0 \end{bmatrix} \xrightarrow{\text{swap } R_1, R_3} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 3 & 7 \\ 2 & 3 & 1 & 3 \end{bmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -3 & 3 & 7 \\ 0 & -1 & 1 & 3 \end{bmatrix}$$

   $$\xrightarrow{\text{swap } R_2, R_3} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & -3 & 3 & 7 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & -3 \\ 0 & -3 & 3 & 7 \end{bmatrix}$$

   $$\xrightarrow[R_3 + 3R_2]{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & 2 & 6 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -2 \end{bmatrix} \xrightarrow[-\frac{1}{2}R_3]{R_1 + 3R_3,\ R_2 - \frac{3}{2}R_3} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   From the last row, we see that the system is inconsistent. Hence, $\bar{v}$ is not in span(S). ☐

2. (5 pts) Is S linearly independent?

   Solution. S is linearly dependent if there exist scalars c_1, c_2, c_3, not all 0, such that

   $$c_1 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

   We solve this system by finding the reduced row echelon form of the corresponding augmented matrix. Notice that the elementary row operations will be the same as above, except there will always be 0s in the last column.

   $$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \to \cdots \to \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   Thus, the linear system above has an infinite number of solutions, and specifically it has a nonzero solution. Hence, S is not linearly independent. ☐