Math 314 Fall 2007 **Quiz #12** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

Recall that the least squares approximation can be determined using the normal equations

$$A^T A\mathbf{x} = A^T\mathbf{b}.$$

1. (10 pts) Find the least squares approximating line for the points $(1, 5)$, $(2, 2)$, $(3, 2)$.

   Solution. We wish to find the line $y = a + bx$ that is the least squares approximation to the given points. We set

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}, \quad , \mathbf{b} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}.$$

   Ideally we would solve $A\mathbf{x} = \mathbf{b}$, but this is an inconsistent system. Thus, we wish to find $\mathbf{x}$ such that $A\mathbf{x}$ most closely approximates $\mathbf{b}$. This $\mathbf{x}$ is given by the solution to the normal equations $A^T A\mathbf{x} = A^T\mathbf{b}$. We thus compute

$$A^T A = \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}, \quad A^T\mathbf{b} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}.$$

   We solve for $\mathbf{x}$ using Gaussian elimination:

$$\begin{bmatrix} 3 & 6 & 9 \\ 6 & 14 & 15 \end{bmatrix} \xrightarrow[R_2 - 2R_1]{\frac{1}{3}R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2]{R_1 - R_2} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix} \xrightarrow{\frac{1}{3}R_1} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix}.$$

   Hence $y = a + bx = 6 - \frac{3}{2}x$ is the least squares approximating line to the given points. □