Math 314 Fall 2007 **Quiz #10** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

Let W be the subspace of $\mathbb{R}^n$ spanned by

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{w}_2 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \quad \mathbf{w}_3 = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}.$$

1. (7 pts) Find a basis for $W^\perp$.

   Solution. Recall that $(\text{row}(A))^\perp = \text{null}(A)$. Thus, we let A be the 3×3 matrix whose rows are $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ 3 & 4 & -1 \end{bmatrix}.$$

   Then $W = \text{row}(A)$, and so $W^\perp$ is $\text{null}(A)$. Hence, we can find a basis for $W^\perp$ by finding a basis for $\text{null}(A)$.

$$A \xrightarrow[R_3 - 2R_1 - 1R_2]{\text{swap } R_1, R_2} \begin{bmatrix} 1 & 0 & -3 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Hence,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} x_3, \quad \text{so a basis for null}(A) \text{ is } \left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \right\}.$$

   □

2. (3 pts) What is the dimension of $W^\perp$? Use this information to determine the dimension of W.

   Solution. Note that $\dim(W^\perp) = \text{nullity}(A) = 1$. Since $\dim(W) + \dim(W^\perp) = n = 3$, $\dim(W) = 2$.

   □