

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

1. (5 pts) Compute the reduced row echelon form of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 10 \end{bmatrix}$$

*Solution.*

$$\begin{aligned} \begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 10 \end{bmatrix} &\rightarrow \begin{matrix} R_2 + R_1 \\ R_3 - 3R_1 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 4 & 4 & 10 \end{bmatrix} \rightarrow \frac{1}{2}R_2 \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & \frac{5}{2} \\ 0 & 4 & 4 & 2 \end{bmatrix} \\ &\rightarrow \begin{matrix} R_1 + R_2 \\ R_3 - 4R_2 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & \frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} R_1 + R_2 \\ \text{change } \frac{5}{2} \end{matrix} \begin{bmatrix} 1 & 0 & 2 & \frac{5}{2} \\ 0 & 1 & 1 & \frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

□

2. (5 pts) Using #1, solve the following linear system of equations. If the system is consistent, write the solution in vector form.

$$\begin{cases} x & -y & +z & = & 0 \\ -x & +3y & +z & = & 5 \\ 3x & +y & +7z & = & 10 \end{cases}$$

*Solution.* First note that the corresponding augmented matrix is  $A$  from #1. Since  $\text{rank}(A) = 2 < \#$  of variables, the system is consistent, and the solution has 1 free variable and hence an infinite number of solutions. From the RREF of  $A$ , we have that

$$\begin{cases} x & +2z & = & \frac{5}{2} \\ y & +z & = & \frac{5}{2} \\ 0 & & = & 0 \end{cases}$$

Hence,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2z + \frac{5}{2} \\ -z + \frac{5}{2} \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} z + \begin{bmatrix} \frac{5}{2} \\ \frac{5}{2} \\ 0 \end{bmatrix}.$$

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