Math 314 Fall 2007 **Practice Test #1** Name: ____________________

Show all of your work. Full credit may not be given for an answer alone.

No notes, books, or papers of any other kind may be used during the exam.

You may have any calculator, but **you may NOT use any function except for add, subtract, multiply, divide, exponentiate, and logarithm.** Any infraction of this rule will result in a zero for your test score.

By taking this exam you are agreeing to abide by these rules and the University of Nebraska–Lincoln Academic Integrity Policy.

Signature: ______________________________

Note: This is a test from an old semester, and was designed for one hour. Our test will be designed for 50 minutes. This test also does not cover change of basis, coordinates, or linear transformations, which are topics covered by our test.

1. (16 pts.) Let $A = \begin{bmatrix} 1 & 2 & -3 & 4 & 3 \\ -2 & -4 & 9 & -7 & -3 \\ 1 & 2 & 0 & 5 & 5 \end{bmatrix}$.

   (a) Find the reduced row echelon form of A.

   (b) What are the rank of A and the nullity of A?

   (c) Is the system $A\mathbf{x} = \mathbf{b}$ consistent, where $\mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ -7 \end{bmatrix}$? If the system is consistent, does it have more than one solution?

2. (24 pts.) Determine if each statement is true or false. Justify your answer if you claim the statement is true; if false, explain why the statement is false or provide an example demonstrating that it is false.

   (a) If an $n \times n$ matrix A is invertible, then $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b}$ in $\mathbb{R}^n$.

   (b) A subset of $\mathbb{R}^n$ containing fewer than n vectors must be linearly independent

   (c) There exists a 5×8 matrix with rank 3 and nullity 2.

   (d) The columns of an invertible matrix are linearly independent.

   (e) If S_1 and S_2 are finite subsets of $\mathbb{R}^n$ having equal spans, then $S_1 = S_2$.

   (f) A linear system with fewer equations than variables always has solutions.

3. (10 pts.) Solve the following system of equations. Write the general solution in vector form.

$$\begin{aligned} x_1 - x_2 + x_3 \qquad &= -4 \\ x_1 - x_2 + 2x_3 + 2x_4 &= -5 \\ 3x_1 - 3x_2 + 2x_3 - 2x_4 &= -11 \end{aligned}$$

4. (15 pts.) Let S be the following set of vectors in $\mathbb{R}^3$:

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$$

(a) Is $\mathbf{v} = \begin{bmatrix} 3 \\ 9 \\ 15 \end{bmatrix}$ in the span of S?

(b) Is S linearly independent?

5. (15 pts.) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 5 \\ 1 & 3 & 1 \end{bmatrix}$.

(a) Compute A^{-1}.

(b) Use A^{-1} to solve the matrix equation $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$.

6. (20 pts.) Prove the following statements.

(a) Prove that if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent, then so is $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2\}$.

(b) Let U be an $m \times n$ matrix, and Q an $n \times n$ matrix. Let $V = UQ$. Prove that span $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a subset of span $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n\}$, where $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are the columns of V and $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ are the columns of U.