MATH 412, FALL 2006 - HOMEWORK 14

FINAL EXAM: 7-10PM, Tuesday, December 12.

WARMUP PROBLEMS: Section 6.2: #1, 4. Section 6.3: #1, 3, 16. Section 7.1: #1, 2, 3, 4, 5, 8.

EXTRA PROBLEMS: Section 6.2: #7, 8, 11. Section 6.3: #6, 11, 14. Section 7.1: #10, 20, 22, 24.

WRITTEN PROBLEMS: Do five of the following six (all six if registered for four credits). Due Wednesday, December 6.

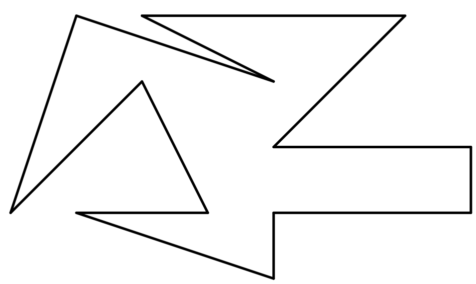
1. Wagner [1937] proved that a graph G is planar if and only if neither K_5 nor $K_{3,3}$ is a minor of G (Definition 6.2.12).

a) Show that deletion and contraction of edges preserve planarity. Conclude from this that Wagner's condition is necessary.

b) Use Kuratowski's Theorem to prove that Wagner's condition is sufficient.

2. Use the Four Color Theorem to prove that every planar graph decomposes into two bipartite graphs.

3. Without using the Four Color Theorem, prove that every outerplanar graph is 3-colorable. Apply this to prove the Art Gallery Theorem: If an art gallery is laid out as a simple polygon with n sides, then it is possible to place $\lfloor n/3 \rfloor$ guards such that every point of the interior is visible to some guard. For $n \geq 3$, construct a polygon that requires $\lfloor n/3 \rfloor$ guards. (Hint: For the construction, use groups of three vertices to build "alcoves" such that no guard can see into more than one alcove.)



4. Let G be a simple graph.

a) Prove that the number of edges in $L(G)$ is $\sum_{v \in V(G)} \binom{d(v)}{2}$.

b) Prove that G is isomorphic to $L(G)$ if and only if G is 2-regular.

5. Use Tutte's 1-factor Theorem to prove that every connected line graph of even order has a perfect matching. Conclude from this that every simple connected graph of even size decomposes into paths of length 2. (Comment: This is another proof of Exercise 1.4.25

6. Let G be a regular graph with a cut-vertex. Prove that $\chi'(G) > \Delta(G)$.