MATH 412, FALL 2006 - HOMEWORK 13

TEST #3: Wed, November 29, from 7-9pm, in 341 Altgeld.

WARMUP PROBLEMS: Section 6.1 #1, 3, 4, 6, 8, 9, 10.

EXTRA PROBLEMS: Section 5.3 #21, 24, 25, 28. Section 6.1 #16, 17, 19, 20, 21, 22, 25, 29, 30, 34, 36

WRITTEN PROBLEMS: Do five of the following six (all six if registered for four credits). Due Friday, December 1.

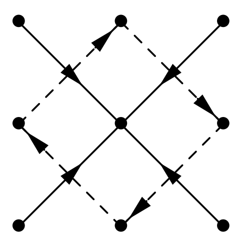
1. Let G be a chordal graph. Use a simplicial elimination ordering of G to prove the following statements.

a) G has at most n maximal cliques, with equality if and only if G has no edges.

b) Every maximal clique of G containing no simplicial vertex of G is a separating set.

2. Prove directly (without using the Perfect Graph Theorem) that the complement of a bipartite graph is perfect. (Hint: Apply a previous result about bipartite graphs.)

3. *Directed plane graphs.* Let G be a plane graph, and let D be an orientation of G. The *dual digraph* D^* is an orientation of the dual graph G^* such that when an edge of D is traversed from tail to head, the dual edge in D^* crosses it from right to left. For example, below we show part of D in solid edges: four edges entering a central vertex. If there are four faces incident to this vertex, then the corresponding edges in D^* form the cycle shown in dashed edges.



Prove that if D is strongly connected, then D^* has no cycle, and $\delta^-(D^*) = \delta^+(D^*) = 0$. Use this to prove that if D is strongly connected, then D has a face on which the edges form a clockwise cycle and another face on which the edges form a counterclockwise cycle.

4. For $n \geq 2$, determine the maximum number of edges in a simple outerplane graph with n vertices, giving three proofs.

a) By induction on n.

b) By using Euler's Formula.

c) By adding a vertex in the unbounded face and using Theorem 6.1.23.

5. Let G be a connected 3-regular plane graph in which every vertex lies on one face of length 4, one face of length 6, and one face of length 8.

a) In terms of $n(G)$, determine the number of faces of each length.

b) Use Euler's Formula and part (a) to determine the number of faces of G.

6. Prove that there is no Eulerian plane multigraph G with either property below (Hint: Consider the dual graph.)

a) Every face of G has length 3 and G has a loop.

b) One face of G has length 2 or 4 and the rest have length 3.