MATH 412, FALL 2006 - HOMEWORK 10

WARMUP PROBLEMS: Section 4.3 #3, 4. Do not write up!

EXTRA PROBLEMS: Section 4.3 #7, 8, 13, 16. Do not write up!

WRITTEN PROBLEMS: Do five of the following six (all six if registered for four credits). Due Wednesday, November 1.

1. Use Menger's Theorem to prove that $\kappa(G) = \kappa'(G)$ when G is 3-regular (Theorem 4.1.11).

2. A u, v-*necklace* is a list of cycles $C_1, \ldots, C_k$ such that $u \in C_1$, $v \in C_k$, consecutive cycles share one vertex, and nonconsecutive cycles are disjoint. Use induction on $d(u, v)$ to prove that a graph G is 2-edge-connected if and only if for all $u, v \in V(G)$ there is a u, v-necklace in G.

3. Let G be a 2-connected simple graph. Prove that in every ear decomposition of G, the number of ears (including the initial cycle) is $e(G) - n(G) + 1$.

4. Use network flows to prove the König–Egerváry Theorem ($\alpha'(G) = \beta(G)$ if G is bipartite).

5. A large university with $3k$ academic departments must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. We must use k assistant professors, k associate professors, and k full professors (each professor has only one rank). How can the committee be found? (Hint: Build a network where units of flow correspond to professors chosen for the committee and capacities enforce the various constraints. Explain how to use the network to test whether such a committee exists and to find it if it does.)

6. In the network below, find a flow of maximum value from s to t. Prove that your answer is optimal by using the dual problem, and explain why this proves optimality.