MATH 412, FALL 2006 - HOMEWORK 1

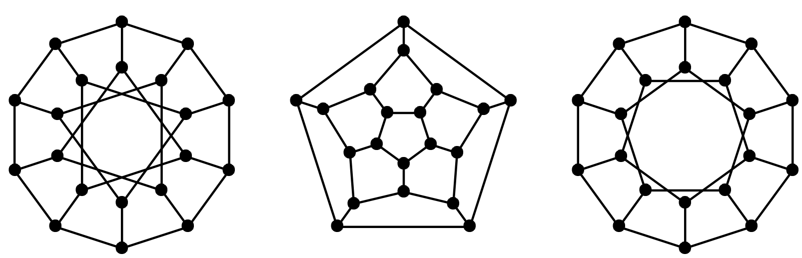
WARMUP PROBLEMS: Section 1.1: #2, 4, 5, 7, 8, 9. Section 1.2: #1, 2, 5. Do not write these up! These problems review basic concepts. Think about how to solve them to clarify your understanding of the material before doing the written homework.

SUGGESTED PROBLEMS: Section 1.1: #14, 18, 20, 24, 27, 35, 37. Section 1.2: #14, 17, 18, 20. Do not write these up! If you have time, think about these for extra practice.

WRITTEN PROBLEMS: Solve five of the following six problems (students registered for four credits must do all six problems). Due Wednesday, August 30 (problem sets are generally due on Wednesdays, with solution sets distributed on Fridays).

Words like “construct”, “show”, “obtain”, “determine”, etc., explicitly state that proof is required. Full credit for solutions to most problems requires proof of the statements made. Use *sentences*; you cannot give a proof without words. Results covered in class can be used without proof (just state them correctly).

1. Determine which pairs of graphs below are isomorphic.



2. Prove that the Petersen graph has no cycle of length 7.

3. Let G be a simple graph with adjacency matrix A and incidence matrix M. Determine what the entries of A^2, MM^T and M^TM mean in terms of the vertices and edges of G. (Hint: The diagonal entries behave differently from the others.)

4. Prove that a self-complementary graph with n vertices exists if and only if n or $n-1$ is divisible by 4. (Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For $n \equiv 1 \pmod 4$, add one vertex to the graph constructed for $n-1$.)

5. Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.

6. Prove that a graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.