Math 412 Fall 2006 Section X14 **Additional Homework #1**

For students registered for 4 credits. Do 8 of the following 10 problems. Due Wed Oct 11 in class.

Read section 8.3 in the textbook. Then do

1. 8.3.21 Assume that if rn is even, then a regular graph on n vertices of degree r exists.
2. 8.3.27
3. 8.3.28
4. 8.3.32 There is a typo in part b): the square root should be $\sqrt{4n(G)-3}$.
5. 8.3.33

Other Problems

6. The Erdős-Gallai condition: Suppose that a sequence $d_1, d_2, \ldots, d_n$ in nonincreasing order is graphic. Prove that for $1 \leq k \leq n$, $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}$ holds, and $\sum d_i$ is even.
7. Let H be a simple graph with n vertices. Prove that if $n > k$ and $e(H) > (k-1)(n-k/2)$, then H contains a subgraph with minimum degree at least k. (*Hint*: Consider the homework that was similar to this.)
8. (a) Prove that if P is a path in a graph G with length less than k, and the endpoints of P each have at least $k/2$ neighbors on P, then G has a cycle with vertex set $V(P)$. (*Hint*: Use the Pigeonhole Principle.)
   (b) Prove that if a graph G has average degree greater than $k-1$, then G contains a path of length k. Show that this fails when the average degree equals $k-1$.
9. A subgraph H of an undirected graph G is a parity subgraph if $\deg_H(v) \equiv \deg_G(v) \pmod 2$ for all $v \in V(G)$. Prove that every spanning tree of G contains a parity subgraph of G. (*Hint*: use a constructive or an inductive proof.)
10. (a) The line digraph $L(D)$ of a digraph D has the arcs of D as vertices, and $e \to f$ in $L(D)$ if and only if there exists a vertex $v \in V(D)$ such that v is the head of e and v is the tail of f. Let D_i be the De Bruijn digraph with string length i. Prove that $D_{i+1} = L(D_i)$.
    (b) Prove that for all i and all $1 \leq k \leq 2^i$, there exists a circuit in D_i of length k. (*Hint*: Use the fact that if H is a subgraph of D, then $L(H)$ is a subgraph of $L(D)$.)