MATH 412, SPRING 2005 - HOMEWORK 9

TEST #2: Monday, March 28. Choose 6-8pm in 159 Altgeld or 7-9pm in 143 Altgeld.

WARMUP PROBLEMS: Section 4.3 #4. Section 5.1 #1, 3, 4, 5, 7, 9, 12, 14. Do not write up!

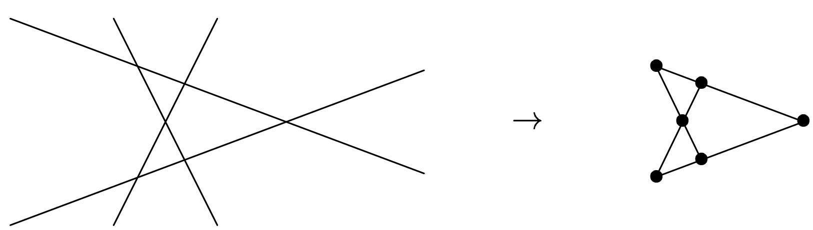
OTHER INTERESTING PROBLEMS: Section 4.3 #7, 8, 13, 16. Section 5.1 #20, 21, 23, 32, 33, 34, 38, 39. Do not write these up!

WRITTEN PROBLEMS: Solve and write five of the following six (all six if registered for four credits). Due Friday, April 1 due to test on Monday, March 28.

1. Use network flows to prove the König–Egerváry Theorem, which states that $\alpha'(G) = \beta(G)$ if G is a bipartite graph.

2. A large university with $3k$ academic departments must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. We must use k assistant professors, k associate professors, and k full professors (each professor has only one rank). How can the committee be found? (Hint: Build a network where units of flow correspond to professors chosen for the committee and capacities enforce the various constraints. Explain how to use the network to test whether such a committee exists and to find it if it does.)

3. Given a set of lines in the plane with no three meeting at a point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. (Hint: This can be solved by using the Szekeres–Wilf Theorem or by using greedy coloring with an appropriate vertex ordering. Comment: The conclusion may fail when three lines are allowed to share a point.)



4. Prove that a graph G is m-colorable if and only if $\alpha(G \,\square\, K_m) \geq n(G)$.

5. Let G be a graph having no induced subgraph isomorphic to P_4. Prove that for every vertex ordering, greedy coloring produces an optimal coloring of G. (Hint: Suppose that the algorithm uses k colors for the ordering $v_1, \ldots, v_n$, and let i be the smallest integer such that G has a clique consisting of vertices assigned colors i through k in this coloring. Prove that $i = 1$.)

6. Prove that $\chi(G) + \chi(\overline{G}) \leq n(G) + 1$. (Hint: Use induction on $n(G)$.)