MATH 412, SPRING 2005 - HOMEWORK 8

TEST #2: Monday, March 28. Again a choice 6-8pm or 7-9pm.

WARMUP PROBLEMS: Section 4.2 #1, 2, 4. Section 4.3 #3. Do not write up!

OTHER INTERESTING PROBLEMS: Section 4.2 #11, 12, 14, 20, 21, 23, 24, 25. Do not write these up!

WRITTEN PROBLEMS: Solve and write five of the following six (all six if registered for four credits). Due Wednesday, March 16.

1. Let G be a graph without isolated vertices. Prove that if G has no even cycles, then every block of G is an edge or an odd cycle.

2. For $k \geq 2$, prove that a graph G with at least $k+1$ vertices is k-connected if and only if whenever S and T are subsets of $V(G)$ with $|S| = k-2$ and $|T| = 2$, there is a cycle in G that contains T and avoids S.

3. *Application of ear decompositions.* Let G be a 2-connected graph.

a) Let s and t be vertices in G. Prove that the vertices of G can be linearly ordered so that each vertex outside $\{s, t\}$ has a neighbor that is earlier in the order and a neighbor that is later in the order.

b) Let r be a vertex in a 2-connected graph G. Prove that G has two spanning trees T and T' such that for every $v \in V(G)$, the r, v-paths in T and T' have no common internal vertices. (Hint: Prove that there is a vertex numbering and trees T and T' such that in T the numbers increase along paths from r and in T' they decrease along paths from r, except for r itself.)

4. Prove that every graph with connectivity k and diameter d has at least $k(d-1)+2$ vertices. Prove for all $k, d \in \mathbb{N}$ that this bound cannot be improved.

5. Let X and Y be disjoint sets of vertices in a k-connected graph G. Let $u(x)$ for $x \in X$ and $w(y)$ for $y \in Y$ be nonnegative integers such that $\sum_{x \in X} u(x) = \sum_{y \in Y} w(y) = k$. Prove that G has k pairwise internally disjoint X, Y-paths so that $u(x)$ of them start at x and $w(y)$ of them end at y, for $x \in X$ and $y \in Y$.

6. In the network below, find a flow of maximum value from c to j. Prove that your answer is optimal by using the dual problem, and explain why this proves optimality.