MATH 412, SPRING 2005 - HOMEWORK 7

WARMUP PROBLEMS: Section 3.3 #1, 2, 4, 5. Section 4.1 #1, 3, 4, 5. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 3.3 #7, 10, 16. Section 4.1 #8, 10, 14, 15, 20, 23, 26, 29. Do not write these up!

WRITTEN PROBLEMS: Solve and write five of the following six (all six if registered for four credits). Due Wednesday, March 9.

1. Prove that a tree T has a perfect matching if and only if $o(T-v)=1$ for every $v \in V(T)$.

2. A *Tutte set* in a graph G is a set S such that $o(G-S) > |S|$. Let G be a connected graph of even order. Prove that every vertex of a minimal Tutte set in G has neighbors in at least three components of $G-S$. Conclude that every claw-free graph of even order has a 1-factor, where a *claw-free* graph is a graph not having $K_{1,3}$ as an induced subgraph.

3. Prove that a 3-regular simple graph has a 1-factor if and only if it decomposes into copies of P_4.

4. Let G be a bipartite graph with n vertices. Prove that if $\delta(G) \geq n/3$, then $\kappa(G) = \delta(G)$. Prove that the result is sharp by obtaining an infinite family of examples where the conclusion fails even though $\delta(G) = (n-1)/3$.

5. *$\kappa'(G) = \delta(G)$ for diameter 2.* Let G be a simple graph with diameter 2, and let $[S, \overline{S}]$ be a minimum edge cut with $|S| \leq |\overline{S}|$.

a) Prove that every vertex of S has a neighbor in $\overline{S}$.

b) Use part (a) and Corollary 4.1.13 to prove that $\kappa'(G) = \delta(G)$.

6. Let $[S, \overline{S}]$ be an edge cut. Prove that there is a set of pairwise edge-disjoint bonds whose union (as edge sets) is $[S, \overline{S}]$. (Note: This is trivial if $[S, \overline{S}]$ is itself a bond.)