MATH 412, SPRING 2005 - HOMEWORK 2

WARMUP PROBLEMS: Section 1.2: #6, 9, 10, 11. Section 1.3: #1, 3, 4, 5. Do not write these up! Think about how to solve them to make sure you understand the material before doing the homework.

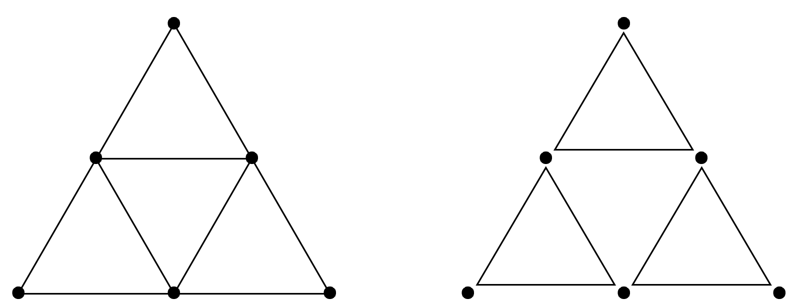
OTHERS OF INTEREST: Section 1.2: #26, 28, 30, 33, 39, 40. Section 1.3: #9, 10, 14, 17, 21, 28, 31, 32, 40, 41. Do not write these up! These are interesting problems (related to what we have discussed) to provide extra practice.

WRITTEN PROBLEMS: Do five of the six problems below (students registered for four credits or honors must do all six problems). Due Wednesday, February 2.

1. Use induction on the number of edges or vertices to prove that absence of odd cycles is a sufficient condition for a graph to be bipartite.

2. Prove that every n-vertex graph with at least n edges contains a cycle.

3. Let G be a connected even graph. For each vertex v, group the edges incident to v into pairs. Since each edge appears in a pair at each endpoint, the pairs link together to form trails. Prove that the pairings produce a decomposition of G into closed trails in this way. (For example, the pairings shown for the graph below produce a decomposition into three trails.) Prove that if the decomposition has more than one trail, then the number of trails can be reduced by changing the pairing at one vertex. Use this procedure to prove that every connected even graph has an Eulerian circuit.



4. *Subgraphs of the Petersen graph.*

a) Prove that every edge of the Petersen graph belongs to exactly four 5-cycles. Use this to count the 5-cycles in the Petersen graph.

b) Determine the maximum number of edges in a bipartite subgraph of the Petersen graph.

5. *Cut-edges in graphs.*

a) Prove that an even graph has no cut-edge.

b) For $j \geq 2$, prove that a j-regular bipartite graph has no cut-edge.

c) For each $k \geq 1$, construct a $2k+1$-regular simple graph having a cut-edge. (Hint: Consider the case $k = 1$ first and then generalize.)

6. Count the 6-cycles in Q_3. Prove that every 6-cycle in Q_k lies in exactly one 3-dimensional subcube. Use this to count the 6-cycles in Q_k for $k \geq 3$.