MATH 412, SPRING 2005 - HOMEWORK 14

WARMUP PROBLEMS: Section 7.2 #2, 4, 5, 12 Section 7.3 #1, 2, 6, 11, 20. Do not write these up! Use them to check your understanding.

OTHERS OF INTEREST: Section 7.2: #6, 7, 8, 18, 20, 26, 30, 34, 35. Section 7.3: #4, 5, 12, 13, 18. Do not write these up!

WRITTEN HOMEWORK: Do five of the following six (all six for four credits). Due Wednesday, May 4.

1. *Hamiltonian vs. Eulerian.*

a) Find a 2-connected non–Eulerian graph whose line graph is Hamiltonian.

b) For a graph G, prove that $L(G)$ is Hamiltonian if and only if G has a closed trail that contains at least one endpoint of each edge.

2. Prove that the cartesian product of two Hamiltonian graphs is Hamiltonian. Conclude that the k-dimensional cube Q_k is Hamiltonian for $k \geq 2$.

3. Let G be a simple graph with vertex degrees $d_1, \ldots, d_n$, indexed so that $d_1 \leq \cdots \leq d_n$. Let $d'_1, \ldots, d'_n$ be the vertex degrees in $\overline{G}$, with $d'_1 \leq \cdots \leq d'_n$. Prove that if $d_i \geq d'_i$ for all $i \leq n/2$, then G has a Hamiltonian path. Conclude that every simple graph isomorphic to its complement has a Hamiltonian path.

4. Let G be a graph that is not a forest. The *circumference* of G is the maximum length of a cycle in G. Prove that G and its Hamiltonian closure have the same circumference.

5. Let G be a plane triangulation.

a) Prove that the dual G^* has a 2-factor.

b) Use part (a) to prove that the vertices of G can be 2-colored so that every face has vertices of both colors. (Hint: Use an idea from the proof of Theorem 7.3.2.)

6. Prove Grinberg's Theorem using Euler's Formula, and use Grinberg's Theorem to prove that the Grinberg graph (shown below) is not Hamiltonian.