MATH 412, SPRING 2005 - HOMEWORK 13

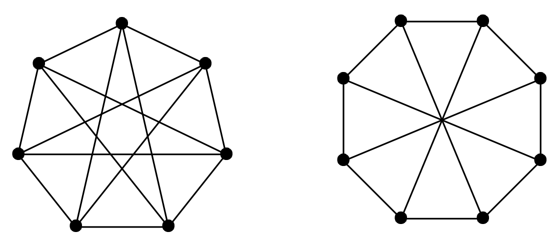
Test #3 on Monday, April 25: 6–8PM in 145 Altgeld Hall or 7–9PM in 143 Altgeld Hall, covering through Homework 12 (coloring of planar graphs, but not crossing number).

WARMUP PROBLEMS: Section 6.3 #4, 16. Section 7.1 #1, 2, 3, 4, 5, 8. Do not write these up! Use them to check your understanding.

OTHERS OF INTEREST: Section 6.3: #20, 24, 25, 26, 28, 34. Section 7.1: #10, 12, 18, 20, 22, 26, 27, 29, 33, 34. Do not write these up!

WRITTEN HOMEWORK: Do five of the following six (all six for four credits). Due Friday, April 29 since Test #3 is Monday, April 25. Homework #14 due Wed., May 4.

1. For $n \geq 5$, let M_n be the graph obtained from the cycle C_n by adding chords joining vertices that are opposite (if n is even) or nearly opposite (if n is odd). Below we show M_7 and M_8; note that M_n is 3-regular if n is even, 4-regular if n is odd. Determine the crossing number of M_n.



2. Suppose that n is odd. Prove that in all drawings of K_n, the parity of the number of pairs of nonincident edges that cross an odd number of times is the same. Use this to show that $\nu(K_n)$ is even when n is congruent to 1 or 3 modulo 8 and is odd when n is congruent to 5 or 7 modulo 8.

3. Let G be a simple graph.

a) Prove that the number of edges in $L(G)$ is $\sum_{v \in V(G)} \binom{d(v)}{2}$.

b) Prove that G is isomorphic to $L(G)$ if and only if G is 2-regular.

4. Use Tutte's 1-factor Theorem to prove that every connected line graph of even order has a perfect matching. Conclude from this that the edges of a simple connected graph of even size can be partitioned into paths of length 2.

5. *Algorithmic proof of Theorem 7.1.7.* Let G be a bipartite graph with maximum degree k. Let f be a proper k-edge-coloring of a subgraph H of G. Let uv be an edge not in H. By using a path alternating in two colors, show that f can be altered and then extended to a proper k-edge-coloring of $H + uv$. Conclude that $\chi'(G) = \Delta(G)$.

6. Let G and H be nontrivial simple graphs. Use Vizing's Theorem to prove that $\chi'(H) = \Delta(H)$ implies $\chi'(G \,\square\, H) = \Delta(G \,\square\, H)$.