# MATH 412, SPRING 2005 - HOMEWORK 11

WARMUP PROBLEMS: Section 5.3 #1, 2, 3, 4. Section 6.1 #1, 3, 4, 6.

OTHERS OF INTEREST: Section 5.3 #5, 7, 8, 11, 14, 19, 20, 23, 24, 28, 29. Section 6.1 #16, 17, 19, 20, 22, 23. Do not write these up!

WRITTEN HOMEWORK: Do five of the following six problems (all six if registered for four credits). Due Wednesday, April 13.

**1.** *Second coefficient of chromatic polynomial.* Let $G$ be a simple graph with $n$ vertices.

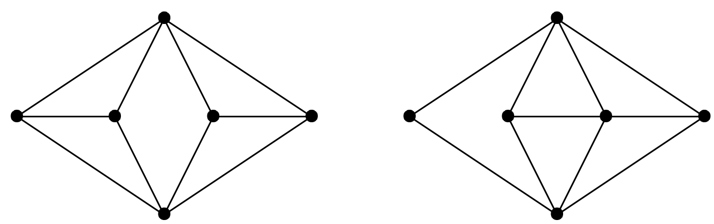
a) Use Proposition 5.3.3 to give a non-inductive proof that the coefficient of $k^{n-1}$ in $\chi(G;k)$ is $-e(G)$. (Hint: Explain how contributions to the coefficient of $k^{n(G)-1}$ arise in computing $\sum_{r=1}^{n} p_r(G)k_{(r)}$.)

b) Use this result and the meaning of $\chi(G;k)$ to determine whether $k^4 - 4k^3 + 4k^2 - k$ is the chromatic polynomial of some graph, and if so, which graph?

**2.** Consider the chromatic polynomials of the graphs below.

a) Without computing them, give a short proof that they are equal.

b) Express this chromatic polynomial as the sum of the chromatic polynomials of two chordal graphs, and use this to give a one-line computation of it.



**3.** Let $G$ be a chordal graph. Use a simplicial elimination ordering of $G$ to prove the following statements.

a) $G$ has at most $n$ maximal cliques, with equality if and only if $G$ has no edges.

b) Every maximal clique of $G$ containing no simplicial vertex of $G$ is a separating set.

**4.** The *Szekeres–Wilf number* of a graph $G$ is $1 + \max_{H \subseteq G} \delta(H)$. Prove that a graph $G$ is chordal if and only if in every induced subgraph the Szekeres–Wilf number equals the clique number.

**5.** For each $n \in \mathbb{N}$ with $n \geq 8$, determine whether there is a simple connected 4-regular planar graph with $n$ vertices.

**6.** Prove that a set of edges in a connected plane graph $G$ forms a spanning tree of $G$ if and only if the duals of the remaining edges form a spanning tree of $G^*$.