MATH 412, SPRING 2005 - HOMEWORK 1

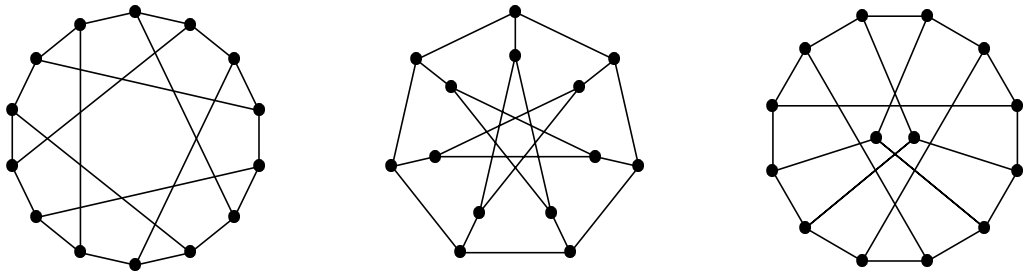
WARMUP PROBLEMS: Section 1.1: #2, 4, 5, 7, 8, 9. Section 1.2: #1, 2, 5. Do not write these up! Just think about how to solve them to make sure you understand the material before working on the written homework.

OTHERS OF INTEREST: Section 1.1: #13, 14, 18, 19, 24, 27, 31. Section 1.2: #14, 17, 18, 20. Do not write these up! If you have time after doing the homework, think about these for extra practice.

WRITTEN PROBLEMS: Solve five of the following six problems (students registered for four credit hours or honors must do all six). Due Wednesday, Jan. 26. Problem sets will usually be due on Wednesdays, with solution sets distributed on Fridays and graded homework returned on Mondays. Some problems have hints in the back of the book; try them first without the hints. Come to the collaborative study sessions or office hours if you have trouble.

Words like "construct", "show", "obtain", "determine", etc., explicitly state that proof is required. Full credit for solutions to most problems requires proof of the statements made. Use *sentences*; you cannot give a proof without words. Results covered in class can be used without proof if stated correctly.

1. Determine which pairs of graphs below are isomorphic.



2. Let G be a graph with girth at least 4 in which every vertex has degree k. Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.

3. Let G be a simple graph with adjacency matrix A and incidence matrix M. Prove that the degree of v_i is the ith diagonal entry in A^2 and in MM^T. What do the entries in position (i, j) of A^2 and MM^T say about G?

4. Decompose the Petersen graph into three connected subgraphs that are pairwise isomorphic. Also decompose it into copies of P_4.

5. Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.

6. Prove that a graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.