Math 250:B1, Quiz #9 | 10 points | Name ________________________
June 19, 2003 | Show all work!

Prove the following statements. Show all steps where you use the properties of determinants, inverses, eigenvalues, transposes, etc.

1. (5 pts.) The transpose of A has the same eigenvalues as A. (*Hint*: Show that A and A^T have the same characteristic polynomials.)

   Solution:

   The eigenvalues of A are exactly the roots of the characteristic polynomial $\det(A - tI)$.

$$\begin{aligned} \det(A - tI) &= \det(A - tI)^T && \text{since } \det B = \det(B^T) \\ &= \det(A^T - (tI)^T) && \text{since } (B + C)^T = B^T + C^T \\ &= \det(A^T - tI) && \text{since } (tI)^T = t(I^T) = tI \end{aligned}$$

   Thus, the characteristic polynomials of A and A^T are the same, and hence A and A^T have the same eigenvalues.

2. (5 pts.; §5.1 #43) If λ is an eigenvalue of an invertible matrix A and $\lambda \neq 0$, then $1/\lambda$ is an eigenvalue of A^{-1}.

   Solution:

   Since λ is an eigenvalue of A, then there is a nonzero vector $\mathbf{v}$ such that

$$\begin{aligned} A\mathbf{v} &= \lambda\mathbf{v}. \\ \text{But then } A^{-1}A\mathbf{v} = \mathbf{v} &= A^{-1}\lambda\mathbf{v} = \lambda(A^{-1}\mathbf{v}), \text{ since } A \text{ is invertible,} \\ \text{and } \frac{1}{\lambda}\mathbf{v} &= A^{-1}\mathbf{v}, \text{ since } \lambda \neq 0. \end{aligned}$$

   Thus, $1/\lambda$ is an eigenvalue of A^{-1}.