

Math 250:B1, Quiz #8
June 17, 2003

10 points
Show all work!

Name _____

$$\text{Let } A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 1 & 4 & -1 \end{bmatrix}.$$

1. (5 pts.) Find a basis for $\text{Col } A$.

Solution:

The reduced row echelon form of A is

$$R = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

The first and second columns of R are pivot columns. Thus, the first and second columns of A are also pivot columns and so form a basis for the column space of A :

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \right\}.$$

2. (5 pts.) Determine the dimensions of $\text{Col } A$, $\text{Row } A$, and $\text{Null } A$.

Solution:

From the reduced row echelon form of A above, $\text{rank } A = 2$ and $\text{nullity } A = 1$. Thus,

$$\begin{aligned} \dim \text{Col } A &= \dim \text{Row } A = \text{rank } A = 2, \text{ and} \\ \dim \text{Null } A &= \text{nullity } A = 1. \end{aligned}$$