Math 250:B1, Quiz #7 10 points Name ________________________
June 16, 2003 Show all work!

Determine whether each subset of $\mathbb{R}^n$ is a subspace. If the set is a subspace, verify the three conditions for being a subspace. If the set is not a subspace, demonstrate which condition is violated.

1. (5 pts.) $V_1 = \left\{ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in \mathbb{R}^2 : u_1 = u_2 \right\}$

   *Solution*:

   $V_1$ is a subspace:

   (a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in V_1$.

   (b) Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be two vectors in $V_1$. Then

   $$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix},$$

   and $u_1 + v_1 = u_2 + v_2$ since $u_1 = u_2$ and $v_1 = v_2$. Thus, $\mathbf{u} + \mathbf{v} \in V_1$.

   (c) Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ be a vector in $V_1$, and let $c$ be a scalar. Then

   $$c\mathbf{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix},$$

   and $cu_1 = cu_2$ since $u_1 = u_2$. Thus, $c\mathbf{u} \in V_1$.

2. (5 pts.; §4.1 #15) $V_2 = \left\{ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in \mathbb{R}^3 : u_1 > u_2 \text{ and } u_3 < 0 \right\}$

   *Solution*:

   $\mathbf{0}$ is not in $V_2$, since the third component must be less than 0. Thus, $V_2$ is not a subspace.