Math 250:B1, Quiz #6 10 points Name ________________________

June 12, 2003 Show all work!

1. (10 pts.; §3.2 #8) Compute $\det \begin{bmatrix} -2 & 1 & -2 \\ 4 & -2 & -1 \\ 0 & 3 & 6 \end{bmatrix}$.

Solution:

This determinant can be calculated using cofactor expansion along any row. We choose the last row since it has a 0 entry.

$$\begin{aligned} \det \begin{bmatrix} -2 & 1 & -2 \\ 4 & -2 & -1 \\ 0 & 3 & 6 \end{bmatrix} &= 0 \cdot (-1)^{3+1} \cdot \det \begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix} + 3 \cdot (-1)^{3+2} \cdot \det \begin{bmatrix} -2 & -2 \\ 4 & -1 \end{bmatrix} \\ &\quad +6 \cdot (-1)^{3+3} \cdot \det \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \\ &= 0(-2+2) - 3(2+8) + 6(4-4) \\ &= -30. \end{aligned}$$

We can also evaluate the determinant by using elementary row operations.

$$\begin{aligned} \det \begin{bmatrix} -2 & 1 & -2 \\ 4 & -2 & -1 \\ 0 & 3 & 6 \end{bmatrix} &= \det \begin{bmatrix} -2 & 1 & -2 \\ 0 & 0 & -5 \\ 0 & 3 & 6 \end{bmatrix} && r_2 + 2r_1 \text{ does not change det} \\ &= -\det \begin{bmatrix} -2 & 1 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & -5 \end{bmatrix} && \text{swapping } r_2 \text{ and } r_3 \text{ changes det by } -1 \\ &= -(-2)(3)(-5) && \text{det of an upper triangular matrix} \\ &= -30. \end{aligned}$$