Math 250:B1, Quiz #5 10 points Name ________________________

June 5, 2003 Show all work!

Determine if each statement is true or false. Justify your answer if you claim the statement is true; if false, explain why the statement is false or provide an example demonstrating that it is false.

1. (3 pts.; §2.3 #1a) Every square matrix is invertible.

   Solution:

   False. The $n \times n$ matrix of all zeros $0_{n\times n}$ is not invertible, since $0_{n\times n}A = 0_{n\times n}$ for any square $n \times n$ matrix A.

2. (3 pts.; §1.6 #1c) If $A = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_k]$ and the matrix equation $A\mathbf{x} = \mathbf{v}$ is inconsistent, then $\mathbf{v}$ does not belong to the span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$.

   Solution:

   True. The vector $\mathbf{v}$ is in the span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ if there exist scalars $c_1, c_2, \ldots, c_k$ such that

   $$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k = v.$$

   Such a collection of scalars would also be a solution to the matrix equation $A\mathbf{x} = \mathbf{v}$. However, since $A\mathbf{x} = \mathbf{v}$ is inconsistent, it has no solutions and so no scalars $c_1, c_2, \ldots, c_k$ can be found to satisfy the above equation. Thus, $\mathbf{v}$ is not in the span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$.

3. (4 pts.; §2.1, #1e) If $\mathbf{v}$ and $\mathbf{w}$ are nonzero vectors in $\mathbb{R}^n$, then $\mathbf{v}\mathbf{w}^T$ is an $n \times n$ matrix of rank 1. (*Hint*: $\mathbf{v}$ and $\mathbf{w}$ are $n \times 1$ column vectors. Try multiplying out $\mathbf{v}\mathbf{w}^T$ and then putting it into reduced row echelon form to determine the rank.)

   Solution:

   True. The product $\mathbf{v}\mathbf{w}^T$ has size $n \times n$ since $\mathbf{v}$ is an $n \times 1$ matrix, and $\mathbf{w}^T$ is a $1 \times n$ matrix. Since $\mathbf{v}$ and $\mathbf{w}$ are nonzero vectors, then $\mathbf{v}\mathbf{w}^T$ has at least one nonzero entry and so its rank must be at least 1. Notice also that each row of $\mathbf{v}\mathbf{w}^T$ is a multiple of $\mathbf{w}^T$. Specifically, the i^{th} row is $v_i\mathbf{w}^T$, where v_i is the i^{th} component of $\mathbf{v}$. It is easy to see that the reduced row echelon form of $\mathbf{v}\mathbf{w}^T$ will thus only have one nonzero row, and so the rank of $\mathbf{v}\mathbf{w}^T$ is 1.