Math 250:B1, Quiz #3 10 points Name ______________________

June 2, 2003 Show all work!

1. (2 pts.) Give the definition of the span of the set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ of vectors of $\mathbb{R}^n$. (*Hint*: The definition should begin "The span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is the set of....")

   Solution: The span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is the set of all linear combinations of $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k$ in $\mathbb{R}^n$. This means that a vector $\mathbf{v}$ is in the span of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ if and only if $\mathbf{v} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k$ for some scalars $c_1, c_2, \ldots, c_k$.

2. (8 pts.; §1.6, #17) Theorem 1.5 states that the following statements about $A_{m\times n}$ are equivalent:

   (a) The span of the columns of A is $\mathbb{R}^m$.

   (b) The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^m$.

   (c) The rank of A is m.

   (d) The reduced row echelon form of A has no zero rows.

   Use Theorem 1.5 to determine if the set

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$$

   spans $\mathbb{R}^3$. Clearly state which part of Theorem 1.5 you are using.

   Solution:

   Let A be the matrix whose columns are the vectors given above:

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 2 \\ -2 & 4 & -2 \end{bmatrix}.$$

   The reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   We need to show that one of parts (b), (c), or (d) is true about A, and then we can conclude by Theorem 1.5 that part (a) is also true. The reduced row echelon form of A has no zero rows, and so part (d) holds for A. Thus, by Theorem 1.5, part (a) is true: the span of the given set of vectors is $\mathbb{R}^3$. From the reduced row echelon form it is also easy to see that the rank of A is 3, and so we have the same result using part (c) of Theorem 1.5.