

Math 250:B1, Quiz #2  
May 29, 2003

10 points  
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Name \_\_\_\_\_

1. (10 pts.; §1.3, #29) Let  $[A \mathbf{b}]$  be the augmented matrix corresponding to the linear system  $A\mathbf{x} = \mathbf{b}$ . Suppose that the reduced row echelon form of  $[A \mathbf{b}]$  is

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Determine if the linear system  $A\mathbf{x} = \mathbf{b}$  is consistent. If the system is consistent, give the general solution in vector form. If the system is inconsistent, state why.

*Solution:* We use the given reduced row echelon form to write down a linear system of equations that is equivalent to  $A\mathbf{x} = \mathbf{b}$ :

$$\begin{array}{rcl} x_1 - 2x_2 & = & 4 \\ x_3 & = & 3 \\ 0 & = & 0 \end{array}$$

Since the reduced row echelon form has no row where the only nonzero entry is in the last column, this system is consistent. Solving for the *basic variables* in terms of the *free variables*, we have

$$\begin{array}{rcl} x_1 & = & 4 + 2x_2 \\ x_2 & \text{free} & \\ x_3 & = & 3. \end{array}$$

Putting this general solution in vector form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$