Math 250:B1, Quiz #11 10 points Name ________________________

June 30, 2003 Show all work!

1. (§6.2, #7) Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \right\}$.

   Using the Gram-Schmidt process, obtain an orthonormal basis for the span of S.

   from the formula sheet:
   Recall that in Gram-Schmidt orthogonalization, you start with a linearly independent set $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_k\} \subseteq \mathbb{R}^n$. Then $S' = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is an orthogonal set where span S' = span S.

$$\begin{aligned} \mathbf{v}_1 &= \mathbf{u}_1 \\ \mathbf{v}_2 &= \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{||\mathbf{v}_1||^2}\mathbf{v}_1 \\ &\vdots \\ \mathbf{v}_k &= \mathbf{u}_k - \frac{\mathbf{u}_k \cdot \mathbf{v}_1}{||\mathbf{v}_1||^2}\mathbf{v}_1 - \ldots - \frac{\mathbf{u}_k \cdot \mathbf{v}_{k-1}}{||\mathbf{v}_{k-1}||^2}\mathbf{v}_{k-1} \end{aligned}$$

   Solution:

   From the Gram-Schmidt process, we have $S' = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$, where

$$\begin{aligned} \mathbf{v}_1 &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \mathbf{v}_2 &= \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} - \frac{6}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} \end{aligned}$$

   Rescaling, we obtain the orthonormal basis

$$\left\{ \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}.$$