Math 250:B1, Quiz #10 | 10 points | Name ____________________

June 26, 2003 | Show all work!

Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \right\}$.

1. (5 pts.) Verify that S is an orthogonal set.

   Solution:

   We need to verify that every pair of vectors in S is orthogonal. Let $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Then

$$\begin{aligned} \mathbf{u}_1 \cdot \mathbf{u}_2 &= 1(-1) + 0(0) + 1(1) &&= 0 \\ \mathbf{u}_1 \cdot \mathbf{u}_3 &= 1(0) + 0(-1) + 1(0) &&= 0 \\ \mathbf{u}_2 \cdot \mathbf{u}_3 &= -1(0) + 0(-1) + 1(0) &&= 0 \end{aligned}$$

   Thus, S is orthogonal.

   We know that since S is an orthogonal set of nonzero vectors, then S is linearly independent. Since S contains three vectors, the span of S must be $\mathbb{R}^3$.

2. (5 pts.) Using dot products and norms, write the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of the vectors in S. (*Hint*: Recall that the orthogonal projection of $\mathbf{v}$ onto a vector $\mathbf{u}$ is $\frac{\mathbf{v} \cdot \mathbf{u}}{||\mathbf{u}||^2}\mathbf{u}$.)

   Solution:

   We have that

$$\begin{aligned} \mathbf{v} &= \frac{\mathbf{v} \cdot \mathbf{u}_1}{||\mathbf{u}_1||^2}\mathbf{u}_1 + \frac{\mathbf{v} \cdot \mathbf{u}_2}{||\mathbf{u}_2||^2}\mathbf{u}_2 + \frac{\mathbf{v} \cdot \mathbf{u}_3}{||\mathbf{u}_3||^2}\mathbf{u}_3 \\ &= \frac{4}{2}\mathbf{u}_1 + \frac{2}{2}\mathbf{u}_2 + \frac{-2}{1}\mathbf{u}_3 \\ &= 2\mathbf{u}_1 + 1\mathbf{u}_2 + -2\mathbf{u}_3 \end{aligned}$$