Math 250:B1, Quiz #1 | 10 points | Name ______________________
May 28, 2003 | Show all work!

1. (5 pts.; §1.1, #43) A matrix A is said to be skew-symmetric if $A^T = -A$. Suppose that A is a skew-symmetric matrix.

   (a) Verify that $a_{ij} = -a_{ji}$.

   (b) Prove that $a_{ii} = 0$. [*Hint*: Use part (a).]

   Solution: We first note that $A = -A^T$. Then

   $$a_{ij} = -[(a^T)_{ij}] = -a_{ji},$$

   verifying (a). Considering the diagonal entries a_{ii},

   $$\begin{aligned} a_{ii} &= -a_{ii}, \\ 2a_{ii} &= 0, \qquad \text{by moving the right side to the left side} \\ a_{ii} &= 0. \end{aligned}$$

2. (5 pts.; §1.2, #13) Compute the following:

   $$\left(\begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 \\ 3 & -3 \end{bmatrix}^T \right) \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

   Your answer should be a single vector with scalar entries.

   Solution:

   $$\begin{aligned} \left(\begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 \\ 3 & -3 \end{bmatrix}^T \right) \begin{bmatrix} 4 \\ 5 \end{bmatrix} &= \left(\begin{bmatrix} 3 & -2 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix} \right) \begin{bmatrix} 4 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 21 \\ 13 \end{bmatrix} \end{aligned}$$