Notes for First Midterm Exam

Math 250:B1, Summer 2003

Rotation Matrices: $A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Matrix Algebra:
$(AB)^T = B^T A^T$
$A(c_1\mathbf{u}_1 + c_2\mathbf{u}_2) = c_1 A\mathbf{u}_1 + c_2 A\mathbf{u}_2$
$(AB)^{-1} = B^{-1}A^{-1}$
$(A^T)^{-1} = (A^{-1})^T$

Algorithm to find A^{-1}: $[A\ I] \xrightarrow{\text{rref}} [I\ A^{-1}]$.

If an augmented matrix $[A\ \mathbf{b}]$ contains a row where the only nonzero entry is in the last column, then $A\mathbf{x} = \mathbf{b}$ is inconsistent.

The rank of A is the number of nonzero rows in the rref of A. The nullity of $A_{m\times n}$ is $n - \text{rank } A$.

$A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b} \in \text{span}\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\}$.

rank of $A_{m\times n}$	number of sols to $A\mathbf{x} = \mathbf{b}$	columns of A	rref of A
m	at least one for every $\mathbf{b} \in \mathbb{R}^m$	spanning set for $\mathbb{R}^m$	every row contains a pivot position
n	at most one for every $\mathbf{b} \in \mathbb{R}^m$	linearly independent	every column contains a pivot position

Performing an elementary row operation on A is the same as multiplying A by the corresponding elementary matrix E.

If R is the rref of $A_{m\times n}$, then there exists an invertible matrix $P_{m\times m}$ such that $PA = R$.

Linear Correspondence Property: Any linear relationship of the columns of A also applies to the columns of rref A, and vice versa.

$A_{n\times n}$ is invertible if and only if rref of A is $I_{n\times n}$.

If A can be put into ref form without using row swaps, then A can be written as LU, where L is a unit lower triangular matrix, and U is an upper triangular matrix. If $U = E_k \cdots E_2 E_1 A$, then $L = E_1^{-1} E_2^{-1} \cdots E_k^{-1}$.