Math 135:04, Quiz #9 — 10 points — Name ______________________

Apr. 4, 2002 — Show all work!

1. (3 pts.) Write “T” or “F” (for True or False) beside each of the mathematical statements below. Each will reward you half-a-point.

   Solution:

F	$\log_b(m+n) = \log_b m + \log_b n$	F	$e^{mn} = e^m e^n$
T	$\ln m + \ln n = \ln mn$	F	$\log_b 0 = 1$
F	$\ln e^x = e^{\ln x} = 1$	T	$\frac{d}{dx} e^{1-x^2} = -2xe^{1-x^2}$

2. (2 pts.) (§5.2, #19) Solve for x:
$$3^{3x-4} = 3^5.$$

   Solution:

   Apply $\log_3$ to both sides:

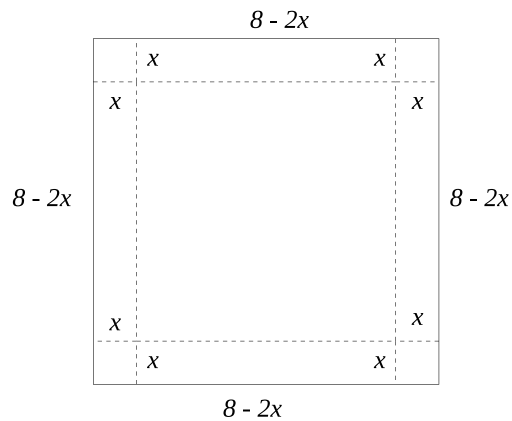
$$\begin{aligned} \log_3 3^{3x-4} &= \log_3 3^5 \\ (3x-4)\log_3 3 &= 5\log_3 3 \\ 3x-4 &= 5 \\ x &= 3. \end{aligned}$$

(Over)

3. (5 pts.) (§4.5, #5) If an *open* box is made from an 8 in. $\times$ 8 in. tin sheet by cutting out identical squares from each corner and bending up the resulting flaps, determine the dimensions of the largest box that can be made. (Here "largest box" means the box with the largest volume.)

Solution:

First we draw and label a figure to aid us:



Step 1, we build the equation for volume $V = lwh$. When we fold up the flaps, we see that h is x. Then l and w are both $8 - 2x$. Now we have

$$V(x) = (8-2x)(8-2x)x = (8-2x)^2 x.$$

Step 2, we find the critical points for $V(x)$:

$$\begin{aligned} V'(x) &= [2(8-2x)(-2)]\, x + (8-2x)^2 \left[1\right] \\ V'(x) &= (8-2x)(-4x + (8-2x)) \\ V'(x) &= (8-2x)(8-6x) \end{aligned}$$

So $V'(x) = 0$ when $x = 4$ or $x = \frac{8}{6} = \frac{4}{3}$.

Step 3, we put bounds on possible x-values (i.e. we need to determine the interval on which we will analyze the function). We use the interval $[0, 4]$ since all other x-values would be nonsense.

Step 4, we tabulate:

x	$V(x)$
0	$(8 - 2 \cdot 0)^2 \cdot 0 = 0$
$\frac{4}{3}$	$(8 - \frac{8}{3})^2 \frac{4}{3} = \frac{1024}{27}$
4	$(8-8)^2 4 = 0$

Finally, we see that the maximum volume occurs when $x = \frac{4}{3}$. We conclude that the dimensions ($l \times w \times h$) of the box are $\frac{16}{3}$in.$\times\frac{16}{3}$in.$\times\frac{4}{3}$in.