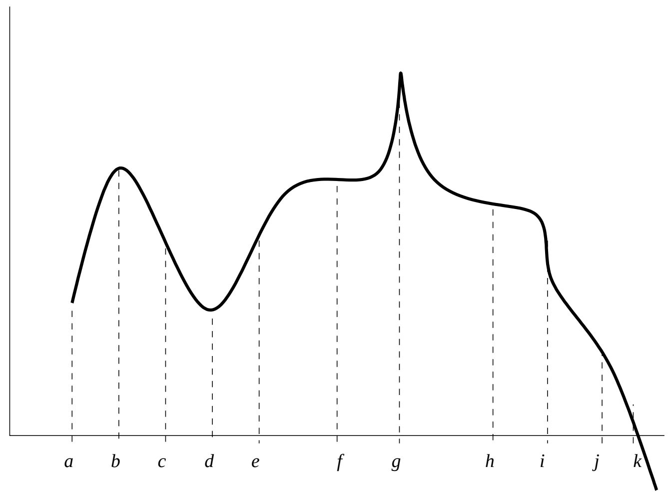
Math 135:04, Quiz #7 | 10 points | Name ________________________
Mar. 14, 2002 | Show all work!

1. (5 pts.) In the graph of $F(x)$ below, for each indicated x-value, match the roman numerals up with the x-values (WARNING: more than one may apply to any particular x-value). We have done the first one for you.

I. This is an inflection point.
II. This is a critical point.
III. This is a local extremum. (NAME THE TYPE!)



Solution:

d — II, III(min), because $F(d) < F(x)$ for x near d.

b — II, because $F'(b) = 0$.
III(max), because $F(b) > F(x)$ for x near b.

e — I, because F changes concavity at e.

f — I, because F changes concavity at f.
II, because $F'(f) = 0$.

g — Not I, because F is concave up on each side of g.
II, because $F'(g)$ is undefined.
III(max), because $F(b) > F(x)$ for x near g.

i — I, because F passes from concave down to concave up at i.
II, because $F'(i)$ is undefined.

(Over)

2. (5 pts) Consider the function $f(x) = \frac{1}{5}x^6 - 2x^4 + 1$.

(a) Is there a critical point at $x = 0$?

Solution:

First we find a few derivatives of f:

$$\begin{aligned} f'(x) &= \frac{6}{5}x^5 - 8x^3 \\ f''(x) &= 6x^4 - 24x^2 \end{aligned}$$

Now, since $f'(0) = 0$, there is a critical point at $x = 0$.

(b) Is there an inflection point at $x = 0$?

Solution:

Notice that $f''(0) = 0$ as well. But this is not enough to tell whether $x = 0$ is an inflection point. To determine the inflection points for f, we must first find all those points where $f''(x) = 0$, then determine if $f''(x)$ changes sign from one side to the other.

$$\begin{aligned} f''(x) &= 0 \\ 6x^4 - 24x^2 &= 0 \\ 6x^2(x^2 - 4) &= 0 \end{aligned}$$

So the solutions are $x = -2, 0,$ or 2.

Next, we choose a test point in each interval to complete the chart below: (We choose $-4, -1, 1,$ and 4.)

interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
$f''(x)$ is	$+$	$-$	$-$	$+$

The chart tells us that there are inflection points for f at -2 and at 2, and that there is *not an inflection point* at $x = 0$.

(c) Can you use the second derivative test to determine if there is a local extremum at $x = 0$? If "Yes," what type is it; if "No," explain why.

Solution:

No, we cannot use the second derivative test. The second derivative test only works if f'' is not zero at the point in question:

If $f'(a) = 0$ and $f''(a) > 0$, then there is a local min at a.
If $f'(a) = 0$ and $f''(a) < 0$, then there is a local max at a.
If $f'(a) = 0$ and $f''(a) = 0$, then the test in inconclusive.