Note: This is a revised version of the quiz as written by A. Lauve and given in class. The first question was not graded, and the second question was graded out of 9 points.

Math 135:04, Quiz #6	10 points	Name ______________________
Mar. 7, 2002	Show all work!	

1. (0 pts.) A biologist is watching a certain population of rodents. She knows that the population $p(t)$ satisfies the equation

$$\frac{dp}{dt} = Np - Dp,$$

where N is the birth rate, D is the death rate, and t is measured in years. When the biologist begins her observations, the initial population size is $10,000$ rodents. Suppose $D = 0.1$ (a terrible disease has struck!) and $N = 0.01$ (the high death rate causing a general state of depression, causing in turn a decrease in amorous feelings).

(a) Using differentials, estimate the rodent population 0.5 years later. (Hint: You know the initial population, and you know how to approximate Δp using differentials.)

Solution:

We recall that $\Delta p \approx dp$, where dp is computed as follows:

$$\begin{aligned} dp &= p'(t)dt \\ &= [Np - Dp]\,dt = [(N-D)p]\,dt \\ &= [-.09 \cdot 10,000]\,(0.5) = -450. \end{aligned}$$

Finally, $p(0.5) - p(0) = \Delta p \approx dp$, so we estimate $p(0.005) \approx dp + p(0) = 9,550$.

(b) Now do the same problem over again, except this time, construct the tangent line $y = mt + b$ to $p(t)$ at time $t = 0$ and give me $y(0.5)$.

Solution:

First, we find m:

$$m = \left.\frac{dp}{dt}\right|_{(0,10,000)} = -900.$$

Next, we find b using the point-slope formula:

$$\begin{aligned} y - y_1 &= m(t - t_1) \\ y - 10,000 &= (-900)(t - 0) \\ y &= -900t + 10,000. \end{aligned}$$

Finally, we compute $y(0.5) = -900(0.5) + 10,000 = 9,550$.

(c) Compare and contrast your results.

Solution:

The answers in (a) and (b) agree because the method of differentials approximates how much the function changes ("Δp") with how much the tangent line changes ("dp"). In part (a) we do this by formula, in part (b) we do this by hand.

(d) Now approximate how many rodents will be left after 50 years, as you did above (using the method in either (a) or (b)).

Solution:

Using the line we have already constructed, we get:

$$\begin{aligned} y(50) &= (-0.09)(50) + 10,000 \\ &= -35,000. \end{aligned}$$

This makes no sense, as population should be positive; still, that's the answer this method of approximation gives.

(e) Suppose you knew that $p(t) = 10,000e^{-0.09t}$. Calculate $p(0.5)$ and $p(50)$ and compare your approximations in (a) and (d) to the true population values. Explain the glaring difference between the two approximations.

Solution:

If $p(t)$ is really as listed above, then $p(0.5) = 10,000e^{-0.045} \approx 9560$, and $p(50) = 10,000e^{-4.5} \approx 111$.

Our approximation in (a) and (b) is off by 10, or about 0.1%. Our approximation in (d) is way, way off. The moral is that a straight line (e.g. a tangent line) cannot approximate a curve for a very large interval. What is a good approximation on the time interval $[0, 0.5]$ is not a good approximation on the time interval $[0, 50]$.

2. (9 pts.) (Derivative Drill) Find y' (dy/dx) for as many of the following functions as you can. **Don't simplify!**

Problem:	*Solution*:
(a) $y = (x^3 + 2x)^{100}$	$y' = 100(x^3 + 2x)^{99}(3x^2 + 2)$
(b) $y = \frac{x+\sqrt{x}}{x^{2/3}}$	$y = x^{1/3} + x^{-1/6}$;
	$y' = \frac{1}{3}x^{-2/3} + \frac{-1}{6}x^{-7/6}$
(c) $y = \frac{x^{2/3}}{x+\sqrt{x}}$	$y' = \frac{\left(\frac{2}{3}x^{-1/3}\right)\left[x+\sqrt{x}\right]-\left(x^{2/3}\right)\left[1+\frac{1}{2}x^{-1/2}\right]}{\left(x+\sqrt{x}\right)^2}$
(d) $y = \frac{f(x)}{g(x)}$	$y' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
(e) $y = f(x)g'(x)$	$y' = f'(x)g'(x) + f(x)g''(x)$
(f) $y^2 = \sqrt{y} + x$	$2yy' = \frac{1}{2}y^{-1/2}y' + 1$;
	$y' = \frac{1}{2y-\frac{1}{2}y^{-1/2}}$
(g) $y = f(g(h(x)))$	$y' = f'(g(h(x)))g'(h(x))h'(x)$
(h) $y = f(g(-x))$	$y' = f'(g(-x))g'(-x)(-1)$
(i) $\left(\sqrt{y} - \frac{1}{2}x\right)^2 = 17y$	$2\left(\sqrt{y} - \frac{1}{2}x\right)\left(\frac{1}{2}y^{-1/2}y' - \frac{1}{2}\right) = 17y'$;
	$y' = \frac{-x\left(\sqrt{y}-\frac{1}{2}x\right)}{17-\left(\sqrt{y}\frac{1}{2}x\right)}$