Math 135:04, Quiz #5 | 10 points | Name ________________
Feb. 28, 2002 | Show all work!

1. (4 pts.) Section 3.6, #34. Consider the equation $(x - y - 1)^3 = x$.

   (a) Use implicit differentiation to find $\frac{dy}{dx}$.

   Solution:

$$\begin{aligned} \frac{d}{dx}\left[(x-y-1)^3\right] &= \frac{d}{dx}[x] \\ 3(x-y-1)^2\left(1 - \frac{dy}{dx}\right) &= 1 \\ \frac{dy}{dx} &= 1 - \frac{1}{3(x-y-1)^2}. \end{aligned}$$

   (b) Find the slope of the line tangent to the graph of the equation at the point $(1, -1)$.

   Solution: The slope of the tangent line is given by $\frac{dy}{dx}$ evaluated at the point $(1, -1)$ (*i.e.*, when $x = 1$ and $y = -1$). Thus we have

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = 1 - \frac{1}{3(1-(-1)-1)^2} = \frac{2}{3}.$$

   (c) Find the equation of the line tangent to the graph at the point $(1, -1)$. Put the line in the form $y = mx + b$.

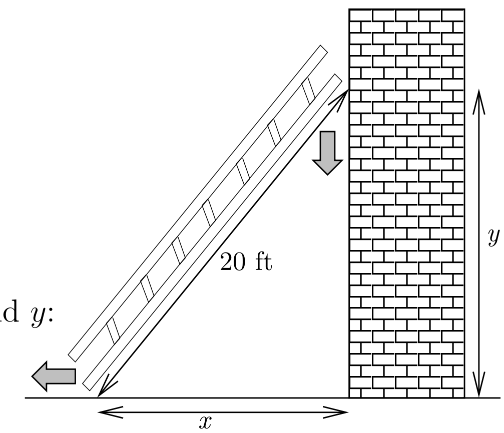
   Solution: The slope m of the tangent line is $2/3$ from above. We solve for the y-intercept b by plugging in the point $(1, -1)$:

$$\begin{aligned} y &= mx + b \\ y &= \frac{2}{3}x + b \\ -1 &= \frac{2}{3}(1) + b \\ -\frac{5}{3} &= b. \end{aligned}$$

   Thus, the equation of the line is $y = \frac{2}{3}x - \frac{5}{3}$.

(Over)

2. (6 pts.) Section 3.6, #60. A 20-ft ladder is leaning up against a wall, as shown in the picture. The ladder begins to slide, so that the top of the ladder is sliding down the wall and the bottom of the ladder is sliding away from the wall.



   (a) Give an equation that relates x and y.

   Solution:
   We use the Pythagorean Theorem to relate x and y:
   $$x^2 + y^2 = 20^2.$$

   (b) Solve for the rate $\frac{dy}{dt}$ at which the top of the ladder is sliding down the wall. (Hint: Use the equation from part 2a.)

   Solution:
   We implicitly differentiate the relation given above, and then solve for $\frac{dy}{dt}$.
   $$\begin{aligned} \frac{d}{dt}\left[x^2 + y^2\right] &= \frac{d}{dt}\left[20^2\right] \\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} &= 0 \\ \frac{dy}{dt} &= -\frac{x}{y}\frac{dx}{dt}. \end{aligned}$$

   (c) How fast is the top of the ladder sliding down the wall at the instant of time when the bottom of the ladder is 12 ft from the wall and sliding away from the wall at the rate of 5 ft/sec? (Hint: Use the relation from part 2a to help solve for unknown values.)

   Solution:
   We need to evaluate $\frac{dy}{dt}$. To do this, we need to know the value of y when $x = 12$ ft. We obtain this from our relation in part 2a:
   $$\begin{aligned} x^2 + y^2 &= 20^2 \\ 12^2 + y^2 &= 20^2 \\ y &= 16 \text{ ft.} \end{aligned}$$
   We now evaluate $\frac{dy}{dt}$ when $x = 12$ ft, $y = 16$ ft, and $\frac{dx}{dt} = 5$ ft/sec:
   $$\frac{dy}{dt} = -\frac{12}{16}(5) = -\frac{15}{4} \text{ ft/sec} = -3.75 \text{ ft/sec}.$$
   Thus, the top of the ladder is sliding down the wall at a rate of 3.75 ft/sec. Note that $\frac{dy}{dt}$ is negative since the height y of the top of the ladder is decreasing.