Math 135:04, Quiz #4 | 10 points | Name ______________________
Feb. 21, 2002 | Show all work!

1. (2 pts.) Let $f(x) = (1 + x^2 + x^3)^4$. Find $f'(x)$.

   Solution: We use the chain rule:

$$\begin{aligned} f'(x) &= 4\left(1 + x^2 + x^3\right)^3 \cdot (1 + x^2 + x^3)' \\ &= 4\left(1 + x^2 + x^3\right)^3 \cdot (2x + 3x^2) \end{aligned}$$

2. (2 pts.) Section 3.3, #53. Suppose $H(x) = g(f(x))$ and $f(2) = 3$, $f'(2) = -3$, $g(3) = 5$, and $g'(3) = 4$. Find $H'(2)$.

   Solution: The chain rule says $H'(x) = g'(f(x)) \cdot f'(x)$. Use the given values:

$$\begin{aligned} H'(2) &= g'(f(2)) \cdot f'(2) \\ &= g'(3) \cdot (-3) = 4 \cdot (-3) = -12. \end{aligned}$$

3. (4 pts.) Let $A(b) = \sqrt{2b^2 - 7}$. Find the following:

   [a] $A'(b)$

   Solution: We use the chain rule:

$$\begin{aligned} A'(b) &= \frac{1}{2}\left(2b^2 - 7\right)^{-\frac{1}{2}} \cdot (4b) \\ &= \frac{2b}{\sqrt{2b^2 - 7}}. \end{aligned}$$

   [b] $A'(2)$

   Solution: We plug in $b = 2$:

$$\begin{aligned} A'(2) &= \frac{2(2)}{\sqrt{2(2)^2 - 7}} \\ &= \frac{4}{1} = 4. \end{aligned}$$

(OVER)

[c] $A''(b)$

Solution: We may compute $A''(b)$ from part [a] using the product rule:

$$\begin{aligned} A''(b) &= (2b)'\left(2b^2-7\right)^{-\frac{1}{2}} + \left[\left(2b^2-7\right)^{-\frac{1}{2}}\right]'(2b) \\ &= 2\left(2b^2-7\right)^{-\frac{1}{2}} + \left[\frac{-1}{2}\left(2b^2-7\right)^{-\frac{3}{2}}(4b)\right](2b) \\ &= 2\left(2b^2-7\right)^{-\frac{1}{2}} - 4b^2\left(2b^2-7\right)^{-\frac{3}{2}}. \end{aligned}$$

[d] $A''(2)$

Solution: We plug in $b = 2$ again:

$$\begin{aligned} A''(2) &= \frac{2}{\sqrt{2(2)^2-7}} - \frac{4(2)^2}{(\sqrt{2(2)^2-7})^3} \\ &= -14. \end{aligned}$$

4. (2 pts.) A hops farmer has a marginal profit function given by $P'(x) = .01x^2 - 20x + 9000$, where x is measured in bushels of hops per month, and $P'(x)$ is valid for $0 < x < 1300$. (Recall that the marginal profit function is supposed to estimate the actual additional profit incurred by increasing production by one unit (bushel). For instance, suppose the manufacturer were currently producing 30 bushels of hops per month. The actual additional profit she'd gain by producing 31 bushels per month is $P(31) - P(30)$. This value is approximated by $P'(30)$.)

[a] Suppose she is currently producing 500 bushels per month. Is it in her best interest to produce 501 bushels per month? Support your answer.

Solution: We need to approximate $P(501) - P(500)$ so we calculate $P'(500)$:

$$\begin{aligned} P'(500) &= .01(500)^2 - 20(500) + 9000 \\ &= 1500. \end{aligned}$$

Since $P'(500)$ is positive, her action would increase her total profit.

Yes, she should produce 501 bushels per year.

[b] A few months later, her farm is producing 1000 bushels of hops per month. Using the same marginal profit function, she wants to determine whether or not further increasing her production will increase her profits. Would you advise her to increase her production to 1001 bushels per month? Support your answer.

Solution: We need to approximate $P(1001) - P(1000)$ so we calculate $P'(1000)$:

$$\begin{aligned} P'(1000) &= .01(1000)^2 - 20(1000) + 9000 \\ &= -1000. \end{aligned}$$

Since $P'(1000)$ is negative, this action would decrease her total profit.

No, she should continue producing 1000 bushels per month.