Math 135:04, Quiz #3 | 10 points | Name ______________________
Feb. 14, 2002 | Show all work!

1. (1 pt.) Let $f(x)$ be a differentiable function defined on the entire real line. State the limit definition of the derivative of f with respect to x.

   Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

2. (3 pts.) Section 2.6, #31. A high-rise construction worker accidentally dropped his screwdriver from a height of 400 ft. After t seconds, the screwdriver had fallen a distance of $s(t) = 16t^2$ ft. How long did it take the screwdriver to hit the ground?

   Solution:

   The screwdriver hit the ground when it had fallen 400 ft.

$$\begin{aligned} s(t) = 16t^2 &= 400 \text{ ft} \\ t &= 5 \text{ sec.} \end{aligned}$$

   What was the velocity of the screwdriver at the time that it hit the ground?

   Solution:

   The velocity of the screwdriver after t seconds is given by the derivative of the distance function.

$$s'(t) = \frac{d}{dt}(16t^2) = 32t.$$

   The velocity of the screwdriver when it hit the ground is the derivative evaluated at $t = 5$ sec.

$$s'(5 \text{ sec}) = 32(5) = 160 \text{ ft/sec}.$$

3. (2 pts.) Suppose that f and g are differentiable functions at $x = 10$, where $f(10) = 1$, $f'(10) = 3$, $g(10) = 2$, and $g'(10) = 5$. Let $h(x) = f(x)g(x)$. Find the value of $h'(10)$ using rules that you know about differentiation.

Solution:

We use the product rule:

$$\begin{aligned} h'(x) &= \frac{d}{dt}\left[f(x)g(x)\right] \\ &= f'(x)g(x) + f(x)g'(x). \end{aligned}$$

Evaluating at $x = 10$, we see that $h'(10) = 3(2) + 1(5) = 11$.

4. (4 pts.) Find the equation of the line tangent to the graph of the function $f(x) = \dfrac{x}{x^2+1} + 1$ at $x = 0$. Put the line in the form $y = mx + b$.

Solution:

The tangent line is of the form $y = mx + b$, where we must determine m and b. The slope m of the line tangent to the graph of $f(x)$ at $x = 0$ is given by the derivative of f evaluated at $x = 0$. First, we calculate $f'(x)$ using the quotient rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[\frac{x}{x^2+1} + 1\right] = \frac{d}{dx}\left[\frac{x}{x^2+1}\right] + 0 \\ &= \frac{\left[\frac{d}{dx}x\right](x^2+1) - x\left[\frac{d}{dx}(x^2+1)\right]}{(x^2+1)^2} \\ &= \frac{x^2+1-x(2x)}{(x^2+1)^2} \\ &= \frac{1-x^2}{(x^2+1)^2}. \end{aligned}$$

Evaluating, we find that $f'(0) = 1 = m$. To find the y-intercept b, we substitute in the point $(0, f(0))$. Note that $f(0) = 1$.

$$\begin{aligned} y &= mx + b \\ y &= 1x + b \\ f(0) = 1 &= 1(0) + b \\ 1 &= b. \end{aligned}$$

Thus, the equation of the line is $y = x + 1$.