Math 135:04, Quiz #2 10 points Name ______________________

Feb. 7, 2002 Show all work!

1. (1 pt.) Evaluate $\lim_{u\to 2} h(u)$, where $h(u) = \dfrac{u^2-4}{u-2}$.

   Solution:
   $$\lim_{u\to 2}\frac{u^2-4}{u-2} = \lim_{u\to 2}\frac{(u+2)(u-2)}{u-2} = \lim_{u\to 2}(u+2) = 4.$$

2. (1 pt.) Suppose that $\lim_{u\to 2} g(u) = 7$. Using $h(u)$ from above, calculate $\lim_{u\to 2}\dfrac{g(u)}{h(u)}$.

   Solution:

   Since $\lim_{u\to 2} h(u) \neq 0$, we may use the rule that says the limit of the quotient is the quotient of the limits:
   $$\lim_{u\to 2}\frac{g(u)}{h(u)} = \frac{\lim_{u\to 2} g(u)}{\lim_{u\to 2} h(u)} = \frac{7}{4}.$$

3. (1 pt.)
   $$\text{Let } f(x) = \begin{cases} 2x-2 & \text{if } x<2 \\ cx^2 & \text{if } x \geq 2 \end{cases}.$$
   Calculate $\lim_{x\to 2^-} f(x)$.

   Solution:
   $$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x-2) = 2.$$

4. (3 pts.) Using the function $f(x)$ above, calculate the value c so that $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$. Show that, with this value of c, $f(x)$ is continuous at $x=2$.

   Solution: First evaluate the right hand limit in terms of c:
   $$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} cx^2 = 4c.$$
   Setting this equal to $\lim_{x\to 2^-} f(x) = 2$, we find that $c = \frac{1}{2}$. For $f(x)$ to be continuous at $x=2$, $\lim_{x\to 2} f(x)$ must exist and must be equal $f(2)$. Since the left hand and right hand limits are equal by our choice of c, $\lim_{x\to 2} f(x)$ exists and is 2. Since $f(2) = \frac{1}{2}(2)^2 = 2$, $f(x)$ is continuous at $x=2$.

5. (3 pts.) Matthew Patrick is a stockbroker who has been watching the stock prices of Widget Enterprises. A client instructed Matthew to sell shares of the stock if the price ever reached *exactly* \$12. The price of the stock yesterday (in dollars) is given by the function

$$p(t) = -\frac{t^3}{64} - \frac{3t}{8} + 20,$$

where t is the number of hours since 9 am ($t = 0$). Assume that Matthew worked the entire day (without breaks) until 5 pm ($t = 8$). Prove that Matthew sold the stock by using the Intermediate Value Theorem. Remember to check all hypotheses of the theorem.

Solution:

First note that $p(t)$ is a polynomial, and thus continuous over the entire real line. We wish to determine if there exists a point c in the interval $[0, 8]$ where $p(c) = 12$. At the endpoints of the interval, $p(0) = 20$ and $p(8) = 9$. Since 12 falls between these values of 20 and 9, and since $p(t)$ is continuous on $[0, 8]$, we can apply the Intermediate Value Theorem to conclude that there exists a point c in the interval $[0, 8]$ where $p(c) = 12$. At time $t = c$, the price of the stock is \$12, and so Matthew sold the stock.

6. (1 pt.) Matthew Patrick has also been watching the stock prices of Lauve & Hartke Answers Unlimited, Inc. A different client instructed Matthew to sell shares of the stock if the price ever reached *exactly* \$100. Before noon, the price was a constant \$67. Due to a breakthrough in problem solving technology, the stock price yesterday jumped at noon to \$123, and then sold the rest of the afternoon for more than \$123. If we wish to show that Matthew sold the stock, why can we *not* use the reasoning used in Problem 5?

Solution:

The function representing the stock price is not continuous, and so the Intermediate Value Theorem does not apply.