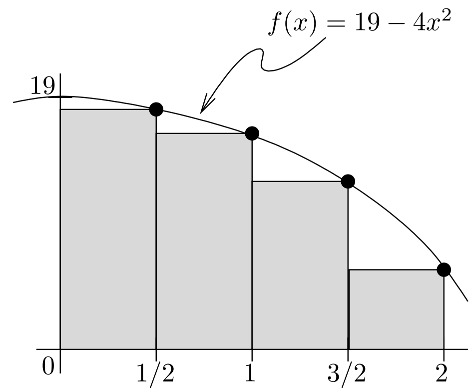
Math 135:04, Quiz #12 — 10 points — Name ______________

May 2, 2002 — Show all work!

1. (a) (3 pts.) Find the Riemann sum for $f(x) = 19 - 4x^2$ on $[0,2]$, using four subintervals of equal length and picking the right endpoint of each subinterval as a representative point. Evaluate the sum to a numeric answer.



Solution:

Since we have four subintervals of $[0,2]$ of equal length, each subinterval has length 1/2. Thus, the subintervals are

$$[0,1/2], \qquad [1/2,1], \qquad [1,3/2], \qquad [3/2,2],$$

and the Riemann sum is

$$\begin{aligned} R &= \frac{1}{2}f\left(\frac{1}{2}\right) + \frac{1}{2}f(1) + \frac{1}{2}f\left(\frac{3}{2}\right) + \frac{1}{2}f(2) \\ &= \frac{1}{2}\left[f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)\right] \\ &= \frac{1}{2}[18+15+10+3] = \frac{1}{2}[46] = 23. \end{aligned}$$

(b) (3 pts.) Evaluate the definite integral $\int_0^2 19 - 4x^2\, dx$.

Solution:

We use the fundamental theorem of calculus:

$$\int_0^2 19 - 4x^2\, dx = F(2) - F(0),$$

where $F(x)$ is any antiderivative of $f(x) = 19 - 4x^2$. We'll choose the antiderivative $F(x) = 19x - 4\frac{x^3}{3}$. Then

$$\begin{aligned} \int_0^2 19 - 4x^2\, dx &= F(2) - F(0) \\ &= \left[19(2) - 4\frac{(2^3)}{3}\right] - [0-0] \\ &= 38 - \frac{32}{3} = \frac{82}{3}. \end{aligned}$$

(Over)

(c) (1 pt.) Suppose that we calculated another Riemann sum for $f(x)$ on $[0, 2]$ using ten subintervals of equal length and picking right endpoints as subinterval representatives. Would this Riemann sum give a better approximation to the definite integral in part 1b than the Riemann sum in part 1a?

Solution:

The definite integral $\int_a^b f(x)\,dx$ is defined as the limit of the Riemann sums as the number of subintervals goes to infinity. Thus, since the Riemann sum described in part 1c has more subintervals than the sum in part 1a, it is a better estimate to the definite integral in part 1b.

2. (3 pts.) §6.5, #22. Evaluate $\int_0^1 \frac{x}{1+2x^2}\,dx$.

Solution:

First, we make the substitution $u = 1 + 2x^2$, noting that $du = 4x\,dx$.

$$\int_0^1 \frac{x}{1+2x^2}\,dx = \int_1^3 \frac{1}{u}\frac{du}{4}.$$

Note that the limits of integration change when we substitute. The new limits are found using the relation $u = 1 + 2x^2$ and plugging in 0 and 1. Now we evaluate the new definite integral using the fundamental theorem of calculus:

$$\begin{aligned}\int_1^3 \frac{1}{u}\frac{du}{4} &= \frac{1}{4}\ln|u|\Big|_1^3 \\ &= \frac{1}{4}[\ln 3 - \ln 1] = \frac{\ln 3}{4}.\end{aligned}$$