Math 135:04, Quiz #11 | 10 points | Name ____________

Apr. 25, 2002 | Show all work!

1. (5 pts) Find the indefinite integrals:

(a) $\int (t-1)t^{3/2}\ dt$ (**Hint:** expand the product first),

Solution: Expanding we have $\int \left(t^{5/2} - t^{3/2}\right) dt$. We integrate each term separately to get

$$\frac{2}{7}t^{7/2} - \frac{2}{5}t^{5/2} + C.$$

(b) (§6.1, #21) $\int \pi\sqrt{t}\ dt$,

Solution: We rewrite the integrand as $\pi t^{1/2}$, then see that the antiderivative is

$$\pi\frac{2}{3}t^{3/2} + C.$$

(c) (§6.2, #11) $\int \frac{x^4}{1-x^5}\ dx$,

Solution: We rewrite as $\frac{1}{-5}\int \frac{(-5)x^4}{1-x^5}\ dx$, which is now in the form $\frac{u'}{u}$, so the answer is

$$\ln|1-x^5| + C.$$

(d) (§6.2, #50) $\int x^3(x^2+1)^{3/2}\ dx$ (**Hint:** let $u = x^2+1$ and hence $du = 2x\ dx$).

Solution: Applying the indicated substitution, we rewrite as follows:

$$\begin{aligned}
\int x^3(x^2+1)^{3/2}\ dx &= \frac{1}{2}\int x^2(x^2+1)^{3/2}2x\ dx \\
&= \frac{1}{2}\int (u-1)u^{3/2}\ du \\
&= \frac{1}{2}\left(\frac{2}{7}(x^2+1)^{7/2} - \frac{2}{5}(x^2+1)^{5/2}\right) + C,
\end{aligned}$$

the last line a result from (a) above.

(Over)

2. (5 pts.) The 'Middle C' string on a piano, which is 60 inches long, is struck. Fixing my gaze on the 15th inch, I know (from my 3rd year physics course) that this point will move up and down with a velocity given by

$$p'(t) = -(2\pi 260)\cos\left(\frac{2\pi}{60}15 - 2\pi 260t\right),$$

where 260Hz is the frequency of Middle C. If I know that $p(0) = 0$, find for me an expression for the position $p(t)$ of the point on the string as a function of time. (**Hint:** When you do indefinite integration, you will get an arbitrary constant C. You will have to use the extra fact I've given about $p(0)$ to solve for the value of C.)

Solution: Simplifying, we get $p'(t) = -(520\pi)\cos\left(\frac{\pi}{2} - 520\pi t\right)$. Antidifferentiating, we get

$$p(t) = \sin\left(\frac{\pi}{2} - 520\pi t\right) + C,$$

but we know $p(0) = 0$, so this gives $0 = \sin\left(\frac{\pi}{2}\right) + C$, or $0 = 1 + C$, so $C = -1$.

Rewriting the equation, we have

$$p(t) = \sin\left(\frac{\pi}{2} - 520\pi t\right) - 1,$$

expressing the position of the point of the string at any time t.