Math 135:04, Quiz #10 | 10 points | Name ________________

Apr. 18, 2002 | Show all work!

1. (4 pts.) Radioactive substances decay exponentially. The amount of the substance present at time t (in days) is given by

$$Q(t) = Q_0 e^{-kt},$$

where Q_0 is the initial amount of the substance at time $t = 0$, and where k is a positive constant. k is known as the *decay constant*. The half-life of Radon 222 is 3.8 days. This means that after 3.8 days, 1/2 of a given quantity of Radon will still be Radon, and the other half will have decayed. Find the decay constant for Radon 222.

Solution:

We know that after 3.8 days, 1/2 of an initial amount Q_0 of Radon will have decayed. Thus,

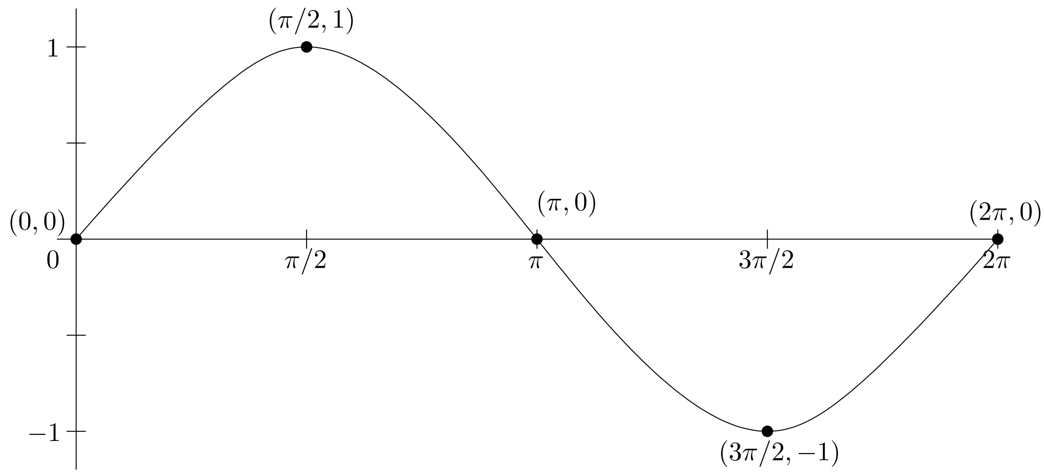
$$\begin{aligned} Q(3.8) = Q_0 e^{-3.8k} &= \frac{1}{2} Q_0 \\ e^{-3.8k} &= \frac{1}{2} \\ -3.8k &= \ln \frac{1}{2} \\ k &= -\frac{\ln \frac{1}{2}}{3.8} \\ k &\approx 0.1824. \end{aligned}$$

Notice that Q_0 cancels out in the first line. The decay constant k is independent of the quantity of Radon and solely depends on the half-life.

(Over)

2. (2 pts.) On the axes provided, draw the curve of $\sin(x)$ on the interval $[0, 2\pi]$. Label the y values on the curve where x is 0, $\pi/2$, π, $3\pi/2$, and 2π.

Solution:



3. (4 pts.) Let $f(x) = \sin^2(2x) = [\sin(2x)]^2$. Find $f'(x)$ and evaluate $f'(\pi/4)$.

Solution:

We need to use the chain rule twice:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left([\sin(2x)]^2\right) \\ &= 2\sin(2x)\frac{d}{dx}\left(\sin(2x)\right) \\ &= 2\sin(2x)[\cos(2x)]\frac{d}{dx}\left(2x\right) \\ &= -4\sin(2x)\cos(2x). \end{aligned}$$

Evaluating, we get

$$\begin{aligned} f'(\frac{\pi}{4}) &= 4\sin(2\frac{\pi}{4})\cos(2\frac{\pi}{4}) \\ &= 4\sin(\frac{\pi}{2})\cos(\frac{\pi}{2}) \\ &= 4(0)(1) \\ &= 0. \end{aligned}$$