Quiz #3
Math 152:18 Calculus II
November 13, 2002

1. (5 pts.) (Sec. 11.7, #13) Determine if $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$ converges or diverges. State which test you use.

   Solution:

   We use the Integral Test. Note that $f(x) = \dfrac{2}{x(\ln x)^3}$ is a continuous, positive, decreasing function on $[2, \infty)$ such that $f(n) = a_n$, and thus the hypotheses of the Integral Test are satisfied.

$$\begin{aligned} \int_2^{\infty} \frac{2}{x(\ln x)^3}\, dx &= \int_{\ln 2}^{\infty} \frac{2}{u^3}\, du \quad \text{by making the substitution } u = \ln x \\ &= -u^{-2}\Big|_{\ln 2}^{\infty} \\ &= (\ln 2)^{-2}. \end{aligned}$$

   Thus, since the integral is convergent, the power series is convergent.

2. (5 pts.) Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

   Solution:

   We use the Ratio Test.

$$\begin{aligned} \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n\to\infty} \left[\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right] \\ &= \lim_{n\to\infty} \frac{x}{n+1} \\ &= 0 < 1 \text{ for all } x \in (-\infty, \infty). \end{aligned}$$

   Thus, the interval of convergence is $(-\infty, \infty)$.