Quiz #1
Math 152:18 Calculus II
September 25, 2002

1. (5 pts.) Integrate $\int \sin^3(x)\cos^2(x)\ dx$.

   Solution:

   We wish to make a u substitution. Since $\sin(x)$ is to an odd power, we keep one $\sin(x)$ to use as du. We then convert the remaining even power of $\sin(x)$ to $\cos(x)$ using the identity $\sin^2(x) = 1 - \cos^2(x)$.

$$\begin{aligned}\int \sin^3(x)\cos^2(x)\ dx &= \int \sin^2(x)\cos^2(x)\,[\sin(x)\ dx] \\ &= \int (1-\cos^2(x))\cos^2(x)\,[\sin(x)\ dx] \\ &= \int \cos^2(x) - \cos^4(x)\,[\sin(x)\ dx]\,.\end{aligned}$$

   Now we substitute $u = \cos(x)$, where $du = -\sin(x)\ dx$:

$$\begin{aligned}\int \cos^2(x) - \cos^4(x)\,[\sin(x)\ dx] &= \int u^2 - u^4\ [-du] \\ &= -\frac{u^3}{3} + \frac{u^5}{5} + C \\ &= -\frac{\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C.\end{aligned}$$

2. (5 pts.) Integrate $\int xe^{2x}\ dx$.

   Solution:

   We use integration by parts with the following substitution:

$$\begin{aligned} u &= x & dv &= e^{2x}\ dx \\ du &= dx & v &= \frac{e^{2x}}{2}\end{aligned}$$

   Thus,

$$\begin{aligned}\int xe^{2x}\ dx &= uv - \int v\ du \\ &= \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\ dx \\ &= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C.\end{aligned}$$