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Summer Mathematics Research Experiences

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Introduction

For the past two decades, summer mathematics research experiences for undergraduates have become increasingly more common. These summer programs are an excellent opportunity for math majors to see if research in math is for them. From my experiences, I found that I really enjoy math research, and have since entered graduate school for mathematics.

I have participated in three summer programs, the REU at Lafayette College ('97), the REU at the University of Minnesota, Duluth, ('98), and the Director's Summer Program (DSP) at the National Security Agency ('99). All three were very worthwhile experiences that gave me different perspectives on a career in mathematics. The following comments are based on my experiences in these three programs, as well the experiences of students I have known in other programs.

The Research

The primary objective of summer research programs is for undergraduate math majors to do research–something usually outside of the classroom experience that comprises the bulk of undergraduate education. If a student does have some research experience at his or her institution, this is usually in the form of a senior thesis–which occurs too late for determining whether or not to attend graduate school. The summer research experience thus fills a necessary part in the professional development of an undergraduate math major.

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That the summer program be a *research* program is very important. If the answers to the problems posed are already known, this is no different than a homework problem. On the other hand, the problems must be attackable. Giving the Riemann Hypothesis to a student to solve in ten weeks will probably lead to frustration, failure, and a sense that real research is simply beyond the abilities of the student.

The matching of a research question to a student is very difficult, and a good method is usually the reason for the success of the longer-running programs. However, no one makes these assignments perfectly. If a problem is solved in a week, or is way too hard and no progress can be made, or even if the problem just does not spark the student's interest, then he should have the option of switching to a new problem. Although a program might concentrate in one specific area (say, graph theory), it should not be focused solely on one question (what is the maximum degree growth rate of the iterated line graph?).

The most important aspect of the problem selection is that the student must feel *ownership* of the problem. The student must really seize the problem, so that he thinks about it when he's brushing his teeth at night and when making breakfast in the morning. If the student doesn't have *his* problem to think about, then he will not experience the excitement of mathematics. I have seen many students doing research, both during the summer and during the academic year, where the research is only a small part of the advisors' larger research plans. These students feel as if they have no control over their work, since *they* are not interested in the research. They are merely cogs in the larger research machine of their advisors.

The need for ownership of a problem does not preclude groups of students working on a problem. Both my group experiences at Lafayette and in the DSP turned out very well. However, the use of a group of students to work on one problem is very risky. If the skills of the group are widely varied, then the members might complement each other quite well and learn a great deal from each other. Or, those less prepared might feel the pressure of keeping up with the group, while those more advanced might feel that the group is holding them back. The group might get along well and enjoy working together. Or there might be personality conflicts that have a debilitating effect on the group's productivity. Group work can be very successful, but it must be undertaken carefully and with an awareness of the hazards.

The advisor plays a crucial role in the summer program. As a more experienced researcher, the advisor can suggest approaches to the problem or find the needed background material. He can critique

proofs that are constructed and can challenge the students to generalize their methods to other problems. In my experience, the advisor has also been a great help when I have gotten stuck and frustrated (something that invariably happens at some point in every research). There the advisors were critical in maintaining a positive outlook and in showing that in research one cannot be easily discouraged.

The Product

Stated alongside the primary goal of gaining research experience is usually a goal of communicating the research that is conducted. One common objective is submitting a paper to a research journal based on the research conducted. Another is presenting the results at a regional or national math conference, as well as giving an end-of-the-summer presentation to all of the participants and advisors in the program. Even though the results of a student's summer research might not be profound, or even complete, it is important to realize that the student's efforts were not wasted. The methods tried and any results obtained, whether partial or complete, is a valuable contribution to the body of knowledge that exists about a problem. Even if the contribution is only a deeper understanding of why a problem is hard, it can be very useful to other mathematicians who later consider the same question.

Most students do not have an opportunity to develop their oral and written mathematical communication skills during their normal undergraduate education. I found that giving periodic talks during the summer on my work was extremely helpful. Not only did I strengthen my speaking skills, but I was also forced to clearly explain what I had been able to prove, what I was currently working on, and what was the sticking point at the moment. The talks also provided a strong incentive to work hard the entire summer.

The preparation of a final report in the form of a research paper is also extremely helpful. In my case, I learned a great deal about the difficulty in writing clearly, and that many, many revisions was the key to overcoming this problem. I also learned the useful skill of writing in LaTeX. Summer programs need to be long enough to allow sufficient background to be developed, research to be conducted, and then a final communication of results obtained. All three goals should be reached by the end of the summer so that a sense of closure can be obtained. After the end of the program, students should continue with the research only if they feel interested, and not because they still need to write the final paper.

The Lifestyle

All three summer programs that I participated in were all very worthwhile from a mathematical standpoint, but they were also a tremendous amount of fun. Eating, sleeping, hanging out with the other students, and doing math–what could be better? The environment that students find themselves in determines how productive they can be mathematically. The availability of comfortable dorms, close proximity to a grocery store, and a large enough stipend to utilize both make for a pleasant summer. Sadly, the low stipends that many summer programs provide discourage many students from participating, since they can make much more working in industry (while learning less).

Socially, having all of the students live together in apartments or dorm rooms that are near each other is a must. The participants are thus able to bond and do things together while not doing mathematics. Students that do not live with the other participants often find themselves not part of the group. Speaking from my experiences, living with a small number of people for several weeks is one of the most enjoyable aspects of the summer programs.

A very successful idea is having planned events, such as picnics or field trips, for the students to get to know each other and the advisors, and to have a break from mathematics. Having the planned events frees the students from attempting to learn the happening places in the region and from needing to organize the adventure. Field trips, such as kayaking and walking on the shore of Lake Superior, are particularly popular at the Duluth REU.

The Duluth program is also notable in that the advisors each year are one or two previous program participants. Several other previous program participants also visit each year to lend their prospectives and insights to the current participants. This concept gives a much greater range of experiences and knowledge for a student to consult than a single advisor, and has the added benefit of providing continuity between successive years of the program. The advisors and the visitors live with the program participants, which adds even more dynamic to the entire social group.

Conclusion

The three summer research programs that I participated in were great experiences. Not only did I learn a lot of math and develop my communication skills, but the experiences confirmed my desire to continue pursuing mathematics. I would strongly encourage any student who is

considering a career in mathematics to participate in a summer research program first.

I would also encourage any faculty member to become involved with such a program. Without the efforts of faculty, these wonderful programs would not exist.

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