In this project, you should hand in the answers to questions 1-9 at the end. Show and explain your work.

As WWI progressed, the British learned to make battle-cruisers with better guns that could shoot artillery shells further and further. By the time of the Battle of Jutland in 1914, a battle cruiser could fire at 16,000 yards (how far is that in miles?). But in the Battle of Jutland, the British navy was not able to blow up a single German ship. Success of the British battle-cruisers had gone down instead of up. There was a lot of speculation as to why this was.

As the range increases, the path of a shell changes. The shell is in the air longer and so air resistance has more time to operate. What other forces operate on a speeding shell? Imagine shooting a shell, at a bit of an upward angle, off the side of a ship. Here’s what it’s trajectory might look like.

Now imagine that your technology has improved and you can shoot the shell further. The new trajectory of the shell will not have exactly the same shape as the old one. This is because air resistance drags the shell down even more. Here’s what the new trajectory might look like.
In the second case, the shell hits the side of the enemy ship at a more oblique angle that it did when it had a shorter range, thus inhibiting its armour piercing abilities. To make the angle of impact against the side of the ship closer to 90°, you would have to decrease the firing distance.

Throughout both wars, newly manufactured shells had to be tested (or “Proved”) to make sure they were good; that is, that they could pierce armour at the distance required and that they were not duds. However testing a shell also destroys it. You can’t test every shell you make, so a few shells were picked from each lot and the quality of these shells was taken as an indicator of the quality of the entire lot of shells.

Here is a memo written by Sir Francis Pridham, head of the British Ordnance Board from 1941 to 1945, which described the process of testing shells.

“The system of Proof was more or less dictated by the shell makers and was such that in the opinion of the Ordnance Board, the only assumption that could make sense of their procedure was that all shell were good shell and that failures were few and due to the machinations of malignant spirits.

The shell were manufactured in Lots, of four hundred, each Lot being subdivided into Sub-lots of one hundred. When a Lot was brought forward by the shell-maker for Proof, two shells were picked out at random from Sub-lot No. 1, to be tested at an armour plate of specified thickness, at a specified striking velocity and at a specified angle of impact. If the first shell succeeded in penetrating the plate whole, the full Lot of (now 399) shell passed into service. If this first shell failed the second was fired. If this was successful, the full lot of (now) 398 were accepted. If the second shell also failed, the Sub-lot was sentenced ‘Reproof’ and the shell maker was given the option of withdrawing the whole lot from Proof or allowing the remaining 3 Sub-lots to proceed to Proof. Needless to say, they generally chose the latter. Proof then commenced with the next Sub-lot of 100, acceptance being governed by the same procedure as that for Sub-lot No. 1.

The Ordnance Board’s Professor of Statistics, having been given the results of Proof firings of the heavy shell then in the fleet, calculated that from 30 percent to 70 percent were probably dud shell, but the data from these Proof firings was insufficient to enable him to give a nearer approximation”

Clearly, shells must be tested to make sure that the manufacturing process is working. But also clearly, we don’t want to test too many shells (since testing the shell destroys it). Is the process that the British Ordnance Board used during WWII adequate?

Here are some problems to try that will help you to answer this question. In these problems, we will assume that the performance of one shell is independent of the performance of any other shell. That is, every shell has the same probability of being a dud- if the first shell is a dud, that doesn’t effect the probability that the second shell is a dud; likewise if the first shell works, that doesn’t effect the probability that the second shell works. You
have talked about this concept of independence in your statistics class and we will discuss it below in the context of manufacturing.

1. **Show that if 50 percent of the shells are duds, using this testing procedure there is a $3/4$ probability that the full lot of 399 or 398 shells will be accepted.**

2. **Show that if 84 percent of the shells are duds, the probability of at least 298 shells being accepted from a Lot is greater than $1/2$. If these shells are accepted, how many of them will be duds?**

This seems really unacceptable, but the problem of testing shells (or anything that must be destroyed to see if it works) is not an easy problem. Let’s explore this a bit more. Suppose that shells are produced in batches of 100 (just to make things a little simpler) and we believe that the proportion of duds can be reduced to 10 percent, though we’re willing to accept anything below 20 percent.

1. **Suppose we test $n$ out of the batch and if any fail we reject the batch.**
Show that in order to reduce the probability of accepting a batch with a 20 percent failure rate to about $1/10$, we must take $n = 10$.

2. Show that if we take $n = 10$ we will reject over 65 percent of the batches in which the proportion of duds is 10 percent.

3. Show that if we take $n = 10$ and suppose that only 10 percent of all of our shell are duds, only 31.5 percent of all of our manufactured shells will actually reach the battleships.
Do you think there’s a good way to test shells? What are the underlying problems? What effect does the assumption of independence have? Do you think that is a sensible assumption? These questions, and others, can be answered on What to Hand In!