1. (5 points each) “Short Answer”, use a single sentence or “True or False” and if false, give a reason why it is false in a single sentence. (If false, 1 point for the answer, 4 points for the reason.)

(a) Short Answer: Why is Geometric Brownian Motion a better model of the stock market than Brownian motion with drift, where the drift parameter is the rate $r$ of market growth?

(b) True or False: The Black-Scholes pricing equation is based on the model that the underlying stock price follows a Brownian Motion.

c) True or False: The Black-Scholes pricing equation values an option by taking the present value of the expected return on the option.

(d) True or False: The closed form solution of the partial differential equation that we call the Black-Scholes formula represents the final word in financial theory.

e) True or False: The volatility of a stock price can be estimated from the Black-Scholes Formula if the option values are known from the market.
(f) True or False: European puts cannot be valued by solving the Black-Scholes equation, only European calls can be valued by solving the Black-Scholes equation.

(g) Short answer: What mathematical property of the Black-Scholes equation allows you to write the formula for the value of a strap (a portfolio consisting of one put and two calls, all with the same strike price) in terms of the value for a call and a put other solutions?
2. (15 points) What is the price of a European put option on a non-dividend-paying when
the stock price is $69, the strike price is $70, the risk-free interest rate is 5% per year
(continuously compounded), the volatility is 35% per year, and the time to maturity is 6
months.
3. (20 points) Use the put-call parity relationship to derive the relationship between

(a) The Delta of European call and the Delta of European put. (The Delta of an option is the rate of change of option value with respect to $S$.)

(b) The Theta of European call and a European put. (The Theta of an option is the rate of change of option value with respect to $t$.)

Show your complete work.
4. (20 points) Find a numerical approximation at \( t = 0.2, 0.4, 0.6, 0.8, 1.0 \) to the solution of the Stochastic Differential Equation:

\[
dX = (1 - X)dt + dW, \quad X(0) = 0.5
\]

(Remark: With some general parameters, this stochastic differential equation is a model of a “mean-reverting process” called the Ornstein-Uhlenbeck process, a useful model in physics and mathematics.) Use \( dt = 0.2 \), and \( N = 100 \) in the table of net totals of randomly generated coin flips below.

<table>
<thead>
<tr>
<th>( j )</th>
<th>( X_j )</th>
<th>((1 - X_j))</th>
<th>( (1 - X_j) , dt )</th>
<th>( dW )</th>
<th>((1 - X_j) , dt + dW )</th>
<th>( X_{j+1} )</th>
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<th>50</th>
<th>60</th>
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<tbody>
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<td>0</td>
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<td>-4</td>
<td>-6</td>
<td>-8</td>
<td>-10</td>
<td>-6</td>
<td>-10</td>
<td>-12</td>
<td>-12</td>
</tr>
</tbody>
</table>
5. (25 points) A company’s cash position, measured in millions of dollars, follows a general Brownian motion with a drift rate of 0.1 per month, and a volatility rate of 0.16 per month. The initial cash position is 2.0. That is, the cash position at time $t$ follows the SDE

$$dX = 0.1 \, dt + 0.16 \, dW$$

$X(0) = 2.0$

Read the problem and the SDE carefully!

(a) What are the probability distributions of the cash position after 1 month, 6 months, and 1 year?

(b) What are the probabilities of a negative cash position at the end of 6 months and one year?