Problem 1

As winner of the lottery, you can choose one of the following prizes:

- $100,000 now
- $113,000 5 years from now
- $2,450 a year forever, starting now
- $10,250 for each of the of the next ten years

Which is the most valuable prize in terms of present value? Assume the interest rate is 2.5% and is compounded continuously (roughly the est available rate at the time of writing this probe.)

\begin{verbatim}
> with(finance);
[amortization, annuity, blackscholes, cashflows, effectiverate, futurevalue, growingannuity, growingperpetuity, levelcoupon, perpetuity, presentvalue, yielddtomaturity]
\end{verbatim}

(a) Has a present value of $100,000

\begin{verbatim}
> r := 0.025;
r := 0.025
> apr := effectiverate(r, infinity);
apr := 0.025315121
> apr_direct := exp(r)-1;
apr_direct := 0.025315121
> PVb := presentvalue(113000, apr, 5);
PVb := 99722.14973
> PVb_direct := 113000/exp(r*5);
PVb_direct := 99722.14999
\end{verbatim}
Problem 2

Interest calculations

A investment of $232 will be worth $312.18 in 2 years. What is the annual interest rate assuming quarterly compounding? Assuming continuously compounded interest?

\[ \text{apr} := \text{fsolve}(312.18 = 232.00 \cdot (1 + r/4)^8, r = 0..1); \]  
\[ \text{apr} := 0.1512092724 \]  

\[ \text{apr} := \text{fsolve}(312.18 = 232.00 \cdot e^{2\cdot r}, r); \]  
\[ \text{apr} := 0.1484212864 \]  

Problem 3

Calculating binomial probabilities

Each day a stock moves up one point or down one point with probabilities $1/4$ and $3/4$ respectively. What is the probability that after 4 days, the stock will have returned to its original price? Assume the daily price fluctuations are independent events.

The stock can return to its original price if and only in the four days, it has increased twice and decreased twice. That is, out of four independent trials we have 2 successes (up days) and consequently 2 failures (down days).

\[ \text{prob} := \text{stats[statevalf, pf, binomiald[4, 1/4]]}(2); \]  
\[ \text{prob} := 0.2109375000 \]  

\[ \text{prob} := \text{binomial(4, 2) \cdot (1/4)^2 \cdot (3/4)^2}; \]  
\[ \text{prob} := \frac{27}{128} \]  

\[ \text{evalf}(%); \]
Problem 4

Calculating binomial and geometric probabilities
Consider a roulette wheel consisting of 38 numbers, $1$ through $36$, $0$ and double $0$. If Bond always bets that the outcome will be one of the numbers $1$ through $12$, what is the probability that Bond will lose his first $5$ bets? What is the probability that his first win will occur on his fourth bet?

\[
\text{prob}_5\text{losses} := \left(\frac{38-12}{38}\right)^5
\]
\[
\text{prob}_5\text{losses} := \frac{371293}{2476099}
\]
\[
\text{evalf}(\%);
\]
\[
0.1499507895
\]
\[
\text{prob}_{\text{win}\_\text{on}\_4} := \left(\frac{38-12}{38}\right)^3\left(\frac{12}{38}\right)
\]
\[
\text{prob}_{\text{win}\_\text{on}\_4} := \frac{13182}{130321}
\]
\[
\text{evalf}(\%);
\]
\[
0.1011502367
\]

Problem 5

Calculating normal probabilities
If $X$ is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute
\begin{enumerate}
\item $\Pr\{X > 5\}$
\item $\Pr\{4 < X < 16\}$
\item $\Pr\{X < 8\}$
\item $\Pr\{X < 20\}$
\item $\Pr\{X > 16\}$
\end{enumerate}

\[
P_{\text{ra}} := 1 - \text{stats[statevalf,cdf,normald[10, 6]]}(5);
\]
\[
P_{\text{ra}} := 0.7976716190
\]
\[
P_{\text{rb}} := \text{stats[statevalf,cdf,normald[10, 6]]}(16) - \text{stats[statevalf,cdf,normald[10, 6]]}(4);
\]
\[
P_{\text{rb}} := 0.6826894922
\]
Problem 6

In $10,000$ independent tosses of a coin, the coin landed heads $5800$ times. Is it reasonable to assume the coin is not fair? Explain.

\begin{align}
\mu &= \frac{10000}{2} \\
\sigma &= \sqrt{10000 \times \frac{1}{2} \times \frac{1}{2}}
\end{align}

\begin{align}
Z &= \frac{5800 - \mu}{\sigma} \\
Z &= 16
\end{align}

It is reasonable to believe the coin is not fair. The coin came up heads more than 16 standard deviations away from the mean, an event that is virtually impossible under the null hypothesis that the coin is fair.

Problem 7

Suppose that $X$ is a random variable having the probability density function

\begin{align}
f(x) &= \begin{cases} R x^{R-1} & \text{for } 0 \leq x \leq 1 \\
0 & \text{elsewhere} \end{cases}
\end{align}

\begin{itemize}
\item Determine the mean $E[X]$.
\item Determine the variance $\Var[X]$.
\item Determine the standard deviation.
\end{itemize}

Assume $R > 0$; density := $R \times x^{R-1}$;
\[ density := R^x \cdot x^{R-1} \quad (7.1) \]

> \texttt{check\_density := int( density, x = 0..1);}
\[ \text{check\_density} := 1 \quad (7.2) \]

> \texttt{mean := int( x*density, x = 0..1);}
\[ \text{mean} := \frac{R^x}{R + 1} \quad (7.3) \]

> \texttt{variance := int( (x - mean)^2 * density, x = 0..1);}
\[ \text{variance} := \frac{R^x}{(R + 1)^2 (R + 2)} \quad (7.4) \]

> \texttt{standard\_deviation := sqrt( variance);}
\[ \text{standard\_deviation} := \sqrt{\frac{R^x}{R + 2}} \quad (7.5) \]

### Problem 8


If $X$ is a uniformly distributed random variable over $(0,1)$, then calculate $E[X^n]$ and $\text{Var}[X^n]$ for $n=1,2,3,\ldots$

> \texttt{assume(n, posint);}

> \texttt{nth\_moment := int( x^n, x = 0..1);}
\[ \text{nth\_moment} := \frac{1}{n + 1} \quad (8.1) \]

> \texttt{nth\_variance := int( (x^n - nth\_moment)^2, x = 0..1);}
\[ \text{nth\_variance} := \frac{n^2}{(n + 1)^2 (1 + 2n)} \quad (8.2) \]

> \texttt{subs( n = 1, nth\_variance);}
\[ \frac{1}{12} \quad (8.3) \]