

# Math 489/889

## Stochastic Processes and Advanced Mathematical Finance

### Final Exam Solutions

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#### Initializations

*with(finance);*

[*amortization, annuity, blackscholes, cashflows, effectiverate, futurevalue, growingannuity, growingperpetuity, levelcoupon, perpetuity, presentvalue, yieldtomaturity*] (1.1)

*with(Statistics)*

[*AbsoluteDeviation, AgglomeratedPlot, AreaChart, BarChart, Bootstrap, BoxPlot, BubblePlot, CDF, CGF, CentralMoment, CharacteristicFunction, ChiSquareGoodnessOfFitTest, ChiSquareIndependenceTest, ChiSquareSuitableModelTest, ColumnGraph, Correlation, CorrelationMatrix, Count, CountMissing, Covariance, CovarianceMatrix, Cumulant, CumulantGeneratingFunction, CumulativeDistributionFunction, CumulativeProduct, CumulativeSum, CumulativeSumChart, DataSummary, Decile, DensityPlot, Discretize, Distribution, ErrorPlot, EvaluateToFloat, Excise, ExpectedValue, ExponentialFit, ExponentialSmoothing, FailureRate, FisherInformation, Fit, FivePointSummary, FrequencyPlot, FrequencyTable, GeometricMean, HarmonicMean, HazardRate, Histogram, Information, InteractiveDataAnalysis, InterquartileRange, InverseSurvivalFunction, Join, KernelDensity, KernelDensityPlot, KernelDensitySample, Kurtosis, Likelihood, LikelihoodRatioStatistic, LineChart, LinearFilter, LinearFit, LogLikelihood, LogarithmicFit, MGF, MLE, MakeProcedure, MaximumLikelihoodEstimate, Mean, MeanDeviation, Median, MedianDeviation, MillsRatio, Mode, Moment, MomentGeneratingFunction, MovingAverage, MovingMedian, MovingStatistic, NonlinearFit, NormalPlot, OneSampleChiSquareTest, OneSampleTTest, OneSampleZTest, OneWayANOVA, OrderByRank, OrderStatistic, PDF, Percentile, PieChart, PointPlot, PolynomialFit, PowerFit, Probability, ProbabilityDensityFunction, ProbabilityFunction, ProbabilityPlot, ProfileLikelihood,*] (1.2)

*ProfileLogLikelihood, QuadraticMean, Quantile, QuantilePlot, Quartile, RandomVariable, Range, Rank, Remove, RemoveInRange, RemoveNonNumeric, Sample, ScatterPlot, Score, Select, SelectInRange, SelectNonNumeric, ShapiroWilkWTest, Shuffle, Skewness, Sort, StandardDeviation, StandardError, StandardizedMoment, SunflowerPlot, Support, SurfacePlot, SurvivalFunction, SymmetryPlot, Tally, TallyInto, Trim, TrimmedMean, TwoSampleFTest, TwoSamplePairedTTest, TwoSampleTTest, TwoSampleZTest, Variance, Variation, WeightedMovingAverage, Winsorize, WinsorizedMean ]*

$Z := \text{RandomVariable}(\text{Normal}(0, 1));$

*\_R*

**(1.3)**

## Problem 1

A stock has a constant volatility of 18% and the risk-free interest rate (compounded continuously) is 6%. What is the value of an option to buy the stock for \$25 in two years time, given the current stock price is \$20?

$S := 20;$   
 $K := 25;$   
 $r := 0.06;$   
 $\sigma := 0.18;$   
 $T_{\text{minust}} := 2;$

20

25

0.06

0.18

2

**(2.1)**

20

25

0.06

0.18

2

**(2.2)**

$\text{evalf}(\text{blackscholes}(S, K, r, T_{\text{minust}}, \sigma));$

1.220977956

**(2.3)**

$d1 := \text{evalf}\left(\frac{\left(\log\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot (T_{\text{minust}})\right)}{\sigma \cdot \sqrt{T_{\text{minust}}}}\right);$

-0.2779069159

**(2.4)**

$$d2 := \text{evalf}\left(\frac{\left(\log\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right) \cdot (T_{\text{minust}})\right)}{\sigma \cdot \sqrt{T_{\text{minust}}}}\right)$$

-0.5324653570 **(2.5)**

$$\text{checkd2} := \text{evalf}(d1 - \sigma \cdot \sqrt{T_{\text{minust}}});$$

-0.5324653571 **(2.6)**

$$\text{Phid1} := \text{CDF}(Z, d1);$$

0.390541907415555667 **(2.7)**

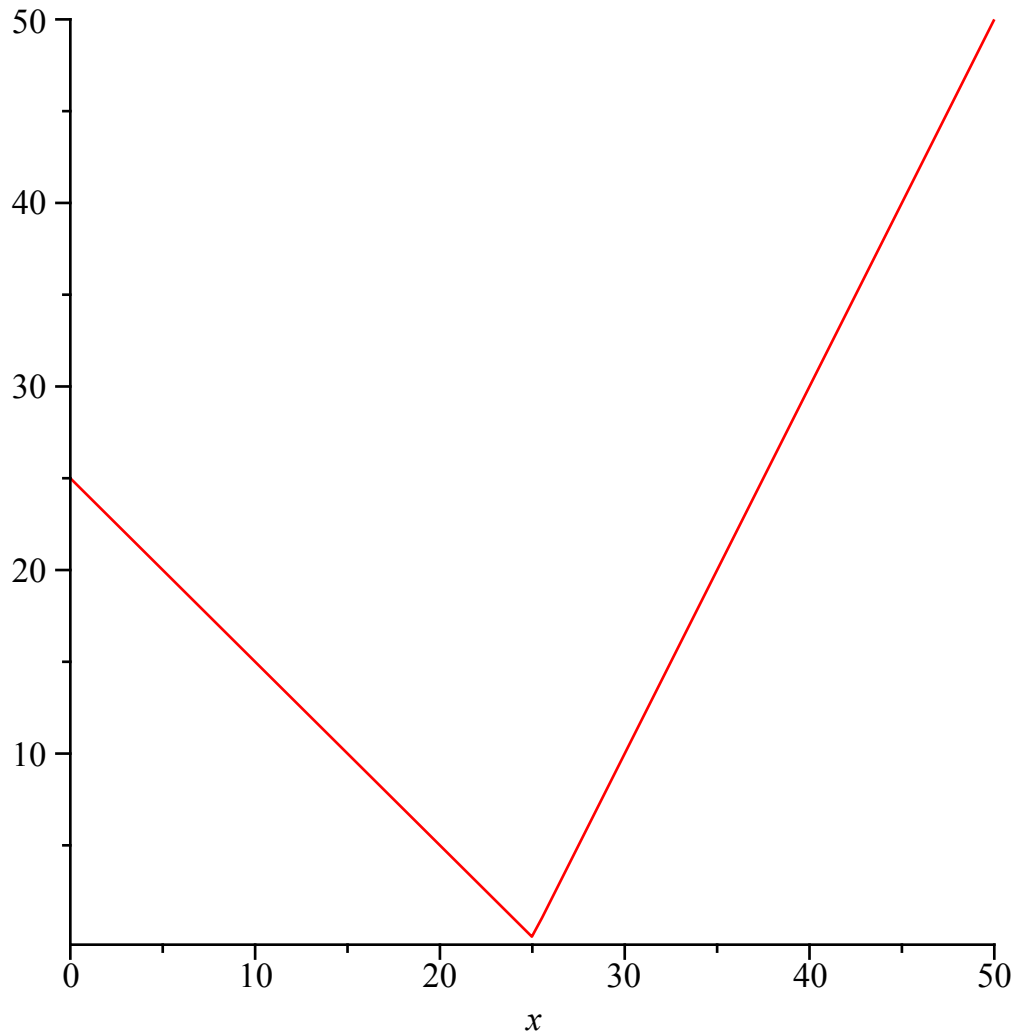
$$\text{Phid2} := \text{CDF}(Z, d2);$$

0.297201864689145112 **(2.8)**

$$V := S \cdot \text{Phid1} - K \cdot \exp(-r \cdot T_{\text{minust}}) \cdot \text{Phid2};$$

1.220977957 **(2.9)**

$\text{plot}(\max(K - x, 0) + 2 \cdot \max(x - K, 0), x = 0..50);$



### Problem 3

$$B := 2 \text{ RandomVariable}(\text{Bernoulli}(0.5)) - 1$$

$$2\_R0 - 1 \tag{3.1}$$

flips := Sample( B, 100);

$$\left[ \begin{array}{l} 1 .. 100 \text{ Vector}_{row} \\ \text{Data Type: float}_8 \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{array} \right] \tag{3.2}$$

S[0] := 0 ;  
**for** i **from** 1 **to** 100 **do**  
    S[i] := S[i - 1] + flips[i] ;  
**od** ;  
[seq( S[10·i], i=0..10 )]  
    [0, -4., -6., -10., -8., -8., -6., -8., -6., -10., -8.] \tag{3.3}

dt := 0.2;

$$0.2 \tag{3.4}$$

Spreadsheet(1)								
	A	B	C	D	E	F	G	H
1	j	X <sub>j</sub>	√X <sub>j</sub>	dW	"dX=(1-X[j])*dt + sqrt(X[j])* dW"	X <sub>j+1</sub> = X <sub>j</sub> + dX		
2	0	0.	0.	-0.6000000000	0.2	0.2		
3	1	0.2	0.4472135955	-0.2000000000	0.07055728090	0.2705572809		
4	2	0.2705572809	0.5201512096	0.2000000000	0.2499187857	0.5204760666		
5	3	0.2499187857	0.4999187791	0.	0.1500162429	0.3999350286		
6	4	0.3999350286	0.6324041655	-0.2000000000	-0.0064678388	0.3934671898		
7	5	0.3934671898						
8								
9								
10								
11								

### Problem 5

r := -0.1;  
σ := 2;  
x<sub>0</sub> := 2.82

$$\begin{array}{l} -0.1 \\ 2 \\ 2.82 \end{array} \tag{4.1}$$

$$\text{evalf}\left(\left[\frac{(x_0 + r \cdot 4)}{\sigma \cdot \sqrt{4}}, \frac{(x_0 + r \cdot 8)}{\sigma \cdot \sqrt{8}}, \frac{(x_0 + r \cdot 12)}{\sigma \cdot \sqrt{12}}\right]\right)$$

**(4.2)**

[0.6050000000, 0.3570889244, 0.2338268591]

$$\text{evalf}\left(\text{CDF}\left(Z, -\frac{(x_0 + r \cdot 4)}{\sigma \cdot \sqrt{4}}\right)\right);$$

**(4.3)**

0.2725894983

$$\text{evalf}\left(\text{CDF}\left(Z, -\frac{(x_0 + r \cdot 8)}{\sigma \cdot \sqrt{8}}\right)\right);$$

**(4.4)**

0.3605126178

$$\text{evalf}\left(\text{CDF}\left(Z, -\frac{(x_0 + r \cdot 12)}{\sigma \cdot \sqrt{12}}\right)\right);$$

**(4.5)**

0.4075596985