

Problem	1	2	Total
Possible	10	10	20
Points			

1. (10 points) Short answer and true or false questions:

- (a) What is the key idea from probability theory and stochastic calculus which is necessary for our derivation of the Black-Scholes partial differential equation?

*The key idea from probability theory and stochastic calculus is that the quadratic variation of Geometric Brownian Motion is  $\sigma^2 S^2 dt$ . Many other ideas from other areas, including economics and finance, are also important, but are more modeling assumptions, rather than key math facts used.*

- (b) True or False: The Black-Scholes pricing equation is based on the model that the underlying stock price follows a Brownian Motion.

*False, the underlying stock price is assumed to follow a Geometric Brownian motion, a different (but related) stochastic process.*

- (c) True or False: The closed form solution of the partial differential equation that we call the Black-Scholes formula represents the final word in financial theory.

*False, it is the starting point and initial ground-breaking idea that stimulated the whole area of mathematical finance.*

- (d) True or False: The volatility of a stock price can be estimated from the Black-Scholes Formula.

*True, that is called the implied volatility.*

- (e) True or False: European puts cannot be valued from the Black-Scholes equation, only European calls.

*False, they can by solving a different terminal value problem, one that we did not choose to solve, but could have.*

2. (10 points) Consider a European call option with a time to expiration 6 months, the underlying stock price is \$42, the exercise price is \$40, the risk-free interest rate is 10% per annum, compounded continuously, and the volatility is 20% per annum. What is the price of such an option?

$d_1 = 0.7693$ ,  $d_2 = 0.6279$ , and  $V = 4.76$