Math 496/889 **Quiz 3** **Name:**________________________

Friday, October 3, 2003 **SSN:**________________________

Problem	1	2	3	4	Total
Possible	5	5	10	5	25
Points					

1. (5 points) Consider a coin-tossing game where a gambler starts with \$10, tosses a fair coin with heads having probability $p = 1/2$ and tails having probability $q = 1/2$. The gambler wins \$1 for every "head" and loses \$1 for every "tail". The gambler is ruined if his fortune reaches \$0 and the gambler is victorious if his fortune reaches \$20. What is the probability of the gambler being victorious?

   The probability of ruin is $q_10 = 1 - S_0/a = 1 - 10/20 = 1/2$. *The probability of victory is* $p_10 = S_0/a = 1/2$

2. (5 points) What is the expected duration (until either ruin or victory) for this game? What is the minimum possible length of a game? What is the maximum possible length of a game?

   The expected duration of the game is $D_10 = S_0(a - S_0) = 10(20 - 10) = 100$. *The minimm possible length of a game is 10 flips, the maximum possible length of a game is infinity (unlimited duration).*

3. (10 points) Use the expected duration of the game from the previous problem as the length of a specific game. What is an estimate of the probability of an excess of 10 or more heads over tails in this game?

   We need $\Pr[S_100 > 10] \approx \Pr[Z > (10 + 1/2)/\sqrt{100}] = \Pr[Z > 1.05] = 0.1469$

4. (5 points) Since an excess of 10 heads over tails would ensure victory, does the estimate from the third question contradict, support, or have no bearing on the answer from the first question? Explain! (Hint: Paths with an excess of 10 heads over tails in n flips are paths which from $S_0 = 10$ at some time cross the level 20 and have $S_n \geq 20$ but we don't care what else happens. Paths which have victory are those paths which touch or cross the level 20 before touching or crossing 0 but we don't care about what happens afterward.)

   I think the easiest way to see this is to consider the set of all paths on 0 *to* 100. *Paths with an excess of* 10 *heads over tails in* 100 *flips are paths which from* $S_0 = 10$ *at some time cross the level* 20 *and have* $S_100 \geq 20$. *Paths which have victory are those paths which touch or cross the level* 20 *before touching or crossing* 0 *but we don't care about what happens afterword. Some paths in the first set may not be in the second set, say if they first cross* 0 *before crossing* 20. *Some paths in the second set may not be in the first set, since they may not have* $S_100 \geq 20$. *The two events are not comparable, and so the probability calculations neither support nor contradict each other.*