Math 496/889 **Quiz 2** **Name:**________________________
Friday, September 19, 2003 **SSN:**________________________

Problem	1	2	Total
Possible	10	10	20
Points			

1. (10 points) Consider a two-time-stage binomial tree example. Each time stage is a year. A stock starts at 100. In each year, the stock can go up by 5% or down by 4%. The interest rate on a \$1 bond is constant at 4.5% each year, compounded continuously. Find the price of a *put option* with exercise price 96, with exercise date at the end of the second year.

   The risk neutral measure is

$$\pi = \frac{\exp(0.045) - 0.96}{1.05 - 0.96} = 0.95587$$

   and $1 - \pi = 0.04413$. *At the three end nodes of the two-stage binomial tree the values of the put option are* 0, 0 *and* 3.84 *respectively. Therefore, the values at each one-stage binomial subtree are:*

$$\begin{aligned} V_I &= \exp(-0.045)[\pi 0 + (1-\pi)0] \\ V_{II} &= \exp(-0.045)[\pi 0 + (1-\pi)3.84] = 0.16202 \\ V_{III} &= \exp(-0.045)[\pi 0 + (1-\pi)0.16202] = 0.00684 \end{aligned}$$

   The option is worth a little less than \$0.01.

2. (10 points) A gambler starts with \$2 and wants to win \$2 more to get to a total of \$4 before being ruined by losing all his money. He plays a coin-flipping game, with a coin that changes with his fortune.

   (a) If the gambler has \$2 he plays with a coin that gives probability $p = 1/2$ of winning a dollar and probability $q = 1/2$ of losing a dollar.

   (b) If the gambler has \$3 he plays with a coin that gives probability $p = 3/4$ of winning a dollar and probability $q = 1/4$ of losing a dollar.

   (c) If the gambler has a dollar he plays with a coin that gives probability $p = 3/4$ of winning a dollar and probability $q = 1/4$ of losing a dollar.

   Use "first step analysis" to write three equations in three unknowns (with two additional boundary conditions) that give the probability that the gambler will be ruined. Solve the equations to find the ruin probability.

   The boundary conditions are $q_4 = 0$, *and* $q_0 = 1$. *The "first-step" equations are:*

$$\begin{aligned} q_2 &= (1/2)q_1 + (1/2)q_3 \\ q_3 &= (3/4)q_4 + (1/4)q_2 \\ q_1 &= (3/4)q_2 + (1/4)q_0 \end{aligned}$$

Subsituting and solving, we find $q_2 = 1/4$, *(and* $q_1 = 7/16$ *and* $q_3 = 1/16$*).*